On improvement to the Shock Propagation Model (SPM) applied to interplanetary shock transit time forecasting

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This paper investigates methods to improve the predictions of Shock Arrival Time (SAT) of the original Shock Propagation Model (SPM). According to the classical blast wave theory adopted in the SPM, the shock propagating speed is determined by the total energy of the original explosion together with the background solar wind speed. Noting that there exists an intrinsic limit to the transit times computed by the SPM predictions for a specified ambient solar wind, we present a statistical analysis on the forecasting capability of the SPM using this intrinsic property. Two facts about SPM are found: (1) the error in shock energy estimation is not the only cause of the prediction errors and we should not expect that the accuracy of SPM to be improved drastically by an exact shock energy input; and (2) there are systematic differences in prediction results both for the strong shocks propagating into a slow ambient solar wind and for the weak shocks into a fast medium. Statistical analyses indicate the physical details of shock propagation and thus clearly point out directions of the future improvement of the SPM. A simple modification is presented here, which shows that there is room for improvement of SPM and thus that the original SPM is worthy of further development.


1. Introduction

CME disturbances at Earth arise from the sheath that arrives in front of the ICME and from the ICME itself. The forecaster is concerned with the arrival of both the ICME shock and the ICME itself [Siscoe and Schwenn, 2006]. Forecasting the onset of CME disturbances from solar or coronal signatures could give 1- to 4-day advance warnings. Empirical models [Schwenn et al., 2000, 2001] and physic-based models [Dryer, 1974; Smith and Dryer, 1990; Fry et al., 2001] are currently used to forecast the arrival time of ICME disturbances, and the latter is expected to reduce the uncertainty of predicted transit time.

Under the spherical symmetric conditions, the non-similarity theory on analytical solutions to the propagation of the blast wave was proposed by Wei [1982] and then was applied to the observational study of shock propagation in interplanetary space [Wei et al., 1983; Wei and Dryer, 1991] and was further applied to the prediction of the geomagnetic disturbances caused by solar wind storms blowing to the Earth [Wei and Cai, 1990; Wei et al., 2002, 2003, 2005; Xie et al., 2006]. Combining the analytical study of the propagation of a blast wave from a point source in a moving, steady-state medium with variable density [Wei, 1982; Wei and Dryer, 1991], with the energy estimation method in the ISPM model [Smith and Dryer, 1990, 1995], a new Shock Propagation Model (SPM) for predicting the arrival time of interplanetary shocks at Earth has been recently presented in the work of Feng and Zhao [2006]. It has a statistical revision according to the dimensionless energy \( E_0 \) (i.e., \( \Delta T(T_0) = 12.789 + 24.692 \log(E_0) + 10.8314(\log(E_0))^2 \)) included in the SPM predictions. We refer to the model without this revision as the original SPM in this paper.

The SPM was applied to the prediction of 165 solar transient events during the periods of January 1979 to October 1989 and February 1997 to August 2002 [Feng and Zhao, 2006]. It proved that the SPM could be practically equivalent to the other three physic-based models that are currently being used to predict the arrival time of ICME shocks, that is, the Shock Time of Arrival model (STOA) [Dryer, 1974], the Interplanetary Shock Propagation Model (ISPM) [Smith and Dryer, 1990], and the Hakamada-Akasofu-Fry version 2 model (HAFv2) [Fry et al., 2001]. The prediction test showed that the relative error of SPM is comparable to that of other three models [Feng and Zhao, 2006]. It implies that taking into account the convective effect of the ambient solar wind is useful to model the real shock propagation. The propagation of shocks generated by
corotating interaction regions (CIRs) and their precursor stream-stream interactions [Dryer, 1998] are not considered in SPM study.

[5] Significant reduction in the error of predicted arrival times will probably not happen until full-up numerical 3D MHD codes become operational that self-consistently integrate the equations of motion of the entire Sun-to-Earth medium [Siscoe and Schwenn, 2006]. Such codes are being constructed and tested but are still in the development stage [Forbes et al., 2006]. So, in this paper, attentions are paid to investigate the SAT predictions with traditional analytical model, that is, the original SPM. In order to give the assessment and improvement of the original SPM, we present a statistical analysis of the prediction errors of it based on the upper limit of the model predicted transit time. We investigate the possibility of improving the original SPM and demonstrate the potential capability of this model in terms of real-time forecasting. We present our data and analysis methods in section 2 and the results and discussion in section 3. Finally a brief summary are delivered in section 4.

2. Data and Methods

2.1. The SAT Prediction With the Original SPM

[6] In the approximated SPM [Wei, 1982], the shock speed $V_s$ is related to the initial shock energy $E_s$ and the ambient solar wind speed $u_0$, which can be written as follows:

$$V_s = \frac{dr}{dt} = u_0 \left[ -2\lambda_1 + \sqrt{(2\lambda_1)^2 + \frac{E_s}{J_0 u_0^2}} + \frac{1}{2J_0} \right],$$  

(1)

where $r$ is heliocentric distance of the shock front position, $\lambda_1$ and $J_0$ are the model constants, and $A$ ($-300 \text{ kg m}^{-1}$) is the density constant at the low corona (in this study, the transition region is ignored and the boundary of the corona set at the surface of the Sun). The nonsimilarity theory was also recalled in the work of Feng and Zhao [2006]. Some details of the derivation of the approximated solutions can be found in that paper. With this approximated relation, the model predicted transit time $TT_m$ of the shock propagating from the solar surface to a position in interplanetary space can be determined by quadrature from the equation:

$$TT_m = \int_{R_e}^{R} \frac{dr}{u_0 \left( a + \frac{E_s}{J_0 u_0^2} \right)},$$  

(2)

where $a = -2\lambda_1$, $b = 1/j_0$, and $c = 4\lambda_2^2 + 1/2J_0$, which are constants in compact form for expression convenience. The solar radius and an arbitrary heliocentric distance specify the integral region ($R_e$, $R$). Equation (2) indicates that a strong shock with larger initial energy will propagate faster than weak shock with lower initial energy and the convective effect of the moving ambient solar wind plays an important role in shock propagation.

[7] Now, we note that the transit time determined by this model has an upper limit for any shock propagating into a moving background medium. According to equation (2), we find that the upper limit, that is, $TT_{\text{max}}$, is reached when the parameter $E_s \rightarrow 0$:

$$TT_{\text{max}} = \int_{R_e}^{R} \frac{dr}{ku_0},$$  

(3)

where, $k = -2\lambda_1 + \sqrt{4\lambda_2^2 + 1/2J_0}$ is the model constant in compact form, and $u_0$ is the ambient solar wind speed. Since any shock which can propagate from the solar surface to a position in interplanetary space must have an initial shock energy $E_s > 0$, the model predicted transit time $TT_m$ must be shorter than the limiting time, that is, $TT_m < TT_{\text{max}}$. This intrinsic limiting property of the original SPM will be applied in the following statistical analysis.

[8] A quadrature form of equation (2) was introduced in the work of Wei [1982], Wei and Dryer [1991], and Feng and Zhao [2006]. In order to calculate $TT_m$ in the most general sense, the revised numeric adaptive quadrature scheme of Gander and Gautschi [2000] is applied here. With the help of this quadrature scheme, $TT_m$ (include $TT_{\text{max}}$) can be computed with the inputs of the solar and interplanetary observations (from data sets that will be described in section 2.2).

[9] The relationships between the transit time predicted by the SPM model $TT_m$ and $u_0$ for different initial shock energy $E_s$ are plotted in Figure 1. The plot of the extreme transit time $TT_{\text{max}}$ versus $u_0$ is also superimposed on it. As illustrated in Figure 1, the $TT_{\text{max}}$ versus $u_0$ plot is almost consistent with the $TT$ versus $u_0$ plot of a weaker shock with $E_s = 1 \times 10^{23}$ erg, which implies that the propagation of weak shocks described by SPM controlled mainly by the movement of ambient solar wind. Up to now, detailed observational study on this convective effect of solar wind has rarely been executed. Here, we present a statistical analysis on this subject with the help of the intrinsic limiting property of the original SPM.

2.2. Data and Methods

[10] We select 137 solar transient events during the period from February 1997 to August 2002, which are taken from Fry et al. [2003] and McKenna-Lawlor et al. [2006]. The events without corresponding IP shock arrival at 1 AU and those with ambiguous relationship between the solar event and the shock at 1 AU are not included here. For each event, the initial shock speed $V_s$, the total shock energy $E_s$, and the background solar wind speed $u_0$ are derived from the solar and interplanetary observations. For example, $V_s$ as the shock speed in corona is estimated from the Type II frequency drift and $E_s$ is estimated with the empirical relation established by Smith and Dryer [1990, 1995]. This energy relation has a form as follows:

$$E_s = CV_s^3 \omega(\tau + D),$$  

(4)

where $C$ and $D$ are assumed to be constant with their values being $0.283 \times 10^{36}$ erg m$^{-3}$ s$^{-2}$ deg$^{-1}$ and 0.52 hours, respectively; an average of angular width $\omega = 60^\circ$ is used as here, and the duration time $\tau$ longer than 2 hours is automatically truncated to 2 hours. We must point out that the total shock energy, $E_s$, is presently unavailable from present solar observations and, indeed, may never be; thus,


Smith and Dryer [1990] were forced to provide an empirical procedure (with the form of equation (4)) to estimate this important parameter. The background speed $u_0$ is the selected as the solar wind speed at 1AU (L1) at the time the solar event. The observed transit time $TT_o$ is the duration time between the begin time (the start of the metric type II radio burst) and arrival time (the time corresponding IP shock observed at 1AU) of each transient event. Both of them are also listed in Table 4 of Feng and Zhao [2006].

Parameters (i.e., $E_o$, $u_0$) are used here as the model inputs for the original SPM. Both the model predicted transit time $TT_m$ and the extreme transit time $TT_{max}$ for each transient event can be computed with equations (2) and (3), respectively, through the numeric quadrature scheme.

Case studies with real observations were used to test the capability of SPM [Feng and Zhao, 2006]. In this work, we further these capability investigations through a statistical analysis of selected data sets classified by the intrinsic limiting property of the original SPM. Theoretically, any shock that can propagate into interplanetary space and be observed at 1AU should have a transit time smaller than $TT_{max}$ since it has an initial energy $E_s > 0$. However, in practice, we find that there are a number of shocks caused by solar transient events that have observed transit time $TT_o$ greater than $TT_{max}$. So, for these events, $TT_m$ will be always smaller than $TT_{max}$, which means that there are the systematic differences between the model predictions $TT_m$ and $TT_o$. In order to investigate these differences, we classified the collected solar transient events into two sets with the criteria: for events with $TT_o \leq TT_{max}$ are of set A, which have rational predictions of SAT, and for events with $TT_o \geq TT_{max}$ are of set B, which have systematic differences in $TT_{max}$. All the 137 events are classified into two subsets, that is, Set A has 86 events (about 62.8%) and Set B has 51 events (about 37.2%). Plots of these data sets are superimposed in Figure 1. The statistical analyses to these data sets are presented in section 3.

3. Results and Discussion

Firstly, for events in Set B, there exists the relation: $TT_o > TT_{max} > TT_m$, and thus the difference between $TT_o$ and $TT_m$ cannot be explained by the uncertainty of the model input parameter $E_s$, which is believed to be main error source of SPM prediction [Wei and Dryer, 1991]. IPS observations and other mathematical method have been applied to reduce the uncertainties in estimation of $E_s$ in previous works [Wei and Cai, 1990; Wei et al., 2002, 2003, 2005; Xie et al., 2006]. In our statistical results, about 37.2% of the total 137 transient events have an exceptional transit time, which cannot be explained by the uncertainties of the model input parameter $E_s$ in original SPM. It suggests that error in shock energy estimation is not the only or even the main cause of the prediction differences and we should not expect that the accuracy problem of SPM can be resolved by an exact shock energy input.

Secondly, as listed in Table 1, statistics of the basic parameters of those transient events in Sets A and B have shown distinct characters. The mean ambient solar wind speed ($u_0$) is 404.5 km/s for events in Set A and 469.2 km/s for events in Set B; the mean initial shock energy ($E_s$) is $2.87 \times 10^{30}$ erg for events in Set A and $1.33 \times 10^{30}$ erg for events in Set B; the averaged dimensionless energy $E_0$ of Set A is $1.559 \times 10^{10}$ and $6.81 \times 10^9$ for events in Set B; but the mean observed transit time ($TT_o$) is 53.1 hours for events in Set A, which is smaller than that of Set B (77.8 hours). According to equation (2), the model predicted transit time $TT_m \propto 1/u_0$ and $TT_m \propto \sqrt{E_0}$, which are referred
to the convective effect of solar wind and the driven effect of solar disturbance described by SPM. Conflicts between the model property and observations indicate that the original SPM is over estimated the convective effect of the moving solar wind, and the initial shock energy is still the main controller of the shock transit time, the higher shock energy makes the shock travel faster. The mean error of model predicted transit time, that is, $(TT_p - TT_m)$, is $-8.1$ hours for Set A, and $21.0$ hours for Set B. It indicates that there are systematic differences in the SPM predictions for both sets. The predictions is on average $8.1$ hours later than observations in Set A and $21.0$ hours ahead of the observations in Set B. Namely, for the strong shocks propagating into a slower ambient solar wind, that is, events in Set A, the SPM prediction of transit time is slightly slow, but for the weak shocks propagating into a faster background medium, that is, events in Set B, the predicted transit time is severely faster than observations.

[15] The causes of these systematic differences in the original SPM consist mainly of three aspects. (1) Due to its excessively simplification, the resulted SPM does not accord with in situ data under the complicated condition, especially for events in Set B, the averaged small dimensionless parameter $\gamma$ is reached $0.629$ (see Table 1), which increased truncation error for model parameters $\lambda = 1 + 2\lambda_1y + 2\lambda_2y^2 + \cdots$ and $J = J_0 (1 + \sigma_1y + \sigma_2y^2 + \cdots)$. Both of them are truncated to the first and zeroth order by $\lambda_1$, $\sigma_1$, and $J_0$, respectively. (2) In the nonsimilarity theory of Wei [1982], the background plasma pressure is assumed to be zero. Ignoring the ambient plasma pressure for weak shock may be another important reason for the systematic difference in the predicted transit time. (3) Ignoring the acceleration of ambient solar wind may be another potential reason for the systematic difference in SPM predictions. The nonsimilarity theory of Wei [1982] takes the back ground medium moving at constants speed, but it has been proved that the solar wind is accelerated from the subsonic state at low corona to its final supersonic state at 1AU and the acceleration exist in the entire solar-terrestrial space [Kojima et al., 2004; Tu et al., 2005].

[16] Physical revision to the current SPM is our next aim. We present a linear correction to the SPM predictions to show the room for improvement of the original SPM. As illustrated in Figure 2, the mean absolute error of model predicted transit time, that is, $(|TT_p - TT_m|)$, is $11.6$ hours for Set A and $21.0$ hours for Set B; the standard deviation of error in model predictions, that is, $\delta = \text{std}(TT_p - TT_m)$, is $12.8$ hours for Set A and $14.0$ hours for Set B. The linear fitting result for Sets A and B can be written as follows:

$$TT_p = 0.658TT_m + 12.854,$$

(5)

and

$$TT_p = 0.989TT_m + 21.666,$$

(6)

equation (5) is a revision for Set A and equation (6) for Set B, where $TT_p$ is the revised prediction. The fitted lines of $TT_o$ to $TT_m$ for Sets A and B are superimposed on Figure 2 as the solid lines, and the deviation of $\pm \delta$, where $\delta = \text{std}(TT_o - TT_m)$, is also plotted as the dashed lines on Figure 2. The mean absolute error of predictions by the revised model, that is, $(|TT_o - TT_m|)$, reach $9.6$ hours for Set A and $10.6$ hours for Set B. A comparison of $TT_m$ and $TT_p$ is also

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**Table 1. Statistics of Basic Parameters of Events in the Two Subdata Sets**

<table>
<thead>
<tr>
<th>Item</th>
<th>Units</th>
<th>Set A</th>
<th>Set B</th>
</tr>
</thead>
<tbody>
<tr>
<td>$E_o$</td>
<td>(km/s)</td>
<td>404.5</td>
<td>469.2</td>
</tr>
<tr>
<td>$V_o$</td>
<td>(km/s)</td>
<td>1042.4</td>
<td>869.2</td>
</tr>
<tr>
<td>$E_o^0$</td>
<td>($\times 10^{15}$ erg)</td>
<td>2.87</td>
<td>1.33</td>
</tr>
<tr>
<td>$V_o^0$</td>
<td>($\times 10^{15}$ #)</td>
<td>1.559</td>
<td>0.681</td>
</tr>
<tr>
<td>$TT_o$</td>
<td>(hour)</td>
<td>53.1</td>
<td>77.8</td>
</tr>
<tr>
<td>$TT_p - TT_m$</td>
<td>(hour)</td>
<td>-8.1</td>
<td>21.0</td>
</tr>
<tr>
<td>$\Delta TT_p$</td>
<td>(hour)</td>
<td>11.6</td>
<td>21.0</td>
</tr>
</tbody>
</table>

*The average dimensionless energy, $E_o = E_o/A/|g|_0$, where $A (=300$ kg m$^{-1}$) is the density constants at low corona.
*The averaged dimensionless variable, $\tau = \text{avg}(V_o/V_s)$.
*Mean error of the model predicted transit time.
*Mean absolute error of the model predicted transit time.
shown in Figure 3. Improvements of the predictions are significant because the linear trends is removed form TT_m, and the slope of the line fitted for TT_o to TT_p is asymptotic to 1. The mean absolute error and the standard deviation of TT_o/C0 TT_p is 10.0 hours and 12.7 hours, respectively, both of them are smaller than that of the original SPM, that is, 15.1 hours and 19.3 hours.

Histograms for event number to error of predicted transit time DT and to the relative error σ = |ΔTT|/TT_o in Figure 4 also shows the improvements. In histogram of event number to ΔTT, both of TT_m and TT_p show Gaussian distributions with a peak around zero, but the peak event number of the TT_p is larger than that of TT_m. In histogram on the right of Figure 4, it shows that the relative absolute error for the TT_p is σ ≤ 10% for 56.2% of the total 137 events, σ ≤ 30% for 86.1%, and σ ≤ 50% for 97.1%, but for TT_m, it is only σ ≤ 10% for 35.8%, σ ≤ 30% for 75.2%, and σ ≤ 50% for 89.6% as listed in Table 2, all three ratios are smaller than that of the revised SPM.

On the basis of the classification of data set by the intrinsic property of SPM, our experimental results have shown that a simple linear revision to the original SPM predictions can lead to a significant improvement for transit time prediction. It also implies that the SPM is potentially valuable to be physically revised for real-time forecasting of shock arrival.

4. Summary and Conclusion

We have attempted to improve the original SPM that is used to forecast the SAT. Through a statistical classification study of the 137 selected solar transient events, we find that there are systematic differences in the SPM predictions. For strong shocks propagating into a slow background medium, the shock transit time given by the SPM is slightly slow, but for weak shocks into a fast medium, the predicted transit time is considerably faster than the observed transit time. The causes of these systematic differences, discussed in this study, these are due to (1) the truncation error of the model parameters λ and J_max, (2) ignoring of ambient plasma pressure before a weak shock, and (3) ignoring of the acceleration of fast solar wind. This last may be the most significant reasons for the systematic differences in SPM.
predictions. As shown by Wu et al. [2004, 2005a], there are complicated interactions between the shocks and/or the discontinuities in the accelerated ambient medium, which will thus affect the propagation of IP shocks. Numerical study to the SAT predictions with 3D MHD codes [Wu et al., 2005b] has also found that the background solar wind speed (BSWS) will affect the SAT at Earth but for the solar disturbances with sufficient large momentum inputs, this affection is paucity. All these facts indicate that the original SPM need to be revised with more realistic physics such as the nonspherical shocks propagated in a nonuniform background medium. A linear revision of the SPM was performed to remedy these differences. The results show that significant improvements to SPM can be gained by this way. It has been found that the mean absolute errors of the four models (STOA, ISPM, HAFV2, and SPM) are all above 12 hours [Feng and Zhao, 2006]. Our revised SPM shows that a mean absolute error of 10 hours can be reached when the systematic differences of SPM predictions are removed. There is room for further improvement and thus the original SPM is worthy of further development. This work also demonstrates the potential value of the non-similarity theory-based SPM in application to the SAT forecasting.

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Table 2. Comparisons of the Relative Errors ($\sigma$)

<table>
<thead>
<tr>
<th>Event Percentage</th>
<th>$\sigma \leq 10%$</th>
<th>$\sigma \leq 30%$</th>
<th>$\sigma \leq 50%$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Original SPM</td>
<td>35.8%</td>
<td>75.2%</td>
<td>89.6%</td>
</tr>
<tr>
<td>Revised SPM</td>
<td>56.2%</td>
<td>86.1%</td>
<td>97.1%</td>
</tr>
</tbody>
</table>

References


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