ROTATIONAL ASYMMETRY OF EARTH’S BOW SHOCK

HU You-Qiu¹, PENG Zhong², WANG Chi²

1 CAS Key Laboratory of Basic Plasma Physics, School of Earth and Space Science, University of Science and Technology of China, Hefei 230026, China
2 State Key Laboratory of Space Weather, Center for Space Science and Applied Research, Chinese Academy of Sciences, Beijing 100190, China

Abstract In terms of global magnetohydrodynamic (MHD) simulations of the solar wind-magnetosphere-ionosphere system, this paper investigates the rotational asymmetry of the Earth’s bow shock with respect to the Sun-Earth line. We are limited to simple cases in which the solar wind is along the Sun-Earth Line, and both the Earth’s magnetic dipole moment and the interplanetary magnetic field (IMF) are perpendicular to the Sun-Earth line. It is shown that even for the case of vanishing IMF strength the bow shock is not rotationally symmetric with respect to the Sun-Earth line: the east-west width of the cross section of the bow shock exceeds the north-south width by about 9%∼11% on the terminator plane (dawn-dusk meridian plane) and its sunward side, and becomes smaller than the north-south width by about 8% on the tailward side of the terminator plane. In the presence of the IMF, the configuration of the bow shock is affected by both the shape of the magnetopause and the anisotropy of fast magnetosonic wave speed. The magnetopause expands outward, being stretched along the IMF, and the extent of its expansion and stretch increases when the IMF rotates from north to south. In the magnetosheath, the fast magnetosonic wave speed is higher in the direction perpendicular to the magnetic field than that in the parallel direction. Therefore, the stretch direction of the magnetopause is perpendicular to the maximum direction of the fast magnetosonic wave speed, and their effects on the bow shock position are exactly opposite. The eventual shape of the bow shock depends on which effect dominates. On the tailward side of the terminator plane, the anisotropy of fast magnetosonic wave speed dominates, so the cross section of the bow shock is wider in the direction perpendicular to the IMF. On the terminator plane and its sunward side, the shape of the bow shock cross section depends on the orientation of the IMF: the bow shock cross section is still wider in the direction perpendicular to the IMF under generic northward or dawn-dusk IMF cases, but it becomes narrower in the direction perpendicular to the IMF instead under generic southward IMF cases. In light of the intimate relationship between the shape of the bow shock and the orientation of the IMF, it is proposed to take the IMF as the datum direction so as to extract the parallel half width \( R_{b\parallel} \) and the perpendicular half width \( R_{b\perp} \) as the scale parameters. In comparison with the commonly used east-west half width \( y_b \) and the north-west half width \( z_b \), these parameters provide a more reasonable description of the geometry of the bow shock. Simulation data show that under the assumption of isotropic orientation of the IMF, the statistical averages of \( y_b/z_b \) and \( R_{b\parallel}/R_{b\perp} \) are both smaller than 1 on the terminator plane, which agrees with relevant observational conclusions.

Key words Earth’s magnetosphere, Interplanetary magnetic field, Bow shock

1 INTRODUCTION

Under the action of the solar wind and interplanetary magnetic field (IMF) the Earth’s magnetic field is confined in a water drop-shaped magnetosphere and a standing shock called bow shock is formed in front of it[1]. The role of the bow shock, which serves as one of important dynamo regions of the solar wind-magnetosphere-ionosphere (SMI) system, has been often ignored for a long time. In recent years, global magnetohydrodynamic (MHD) simulations of the SMI system showed that the bow shock has an important contribution to the region 1 field-aligned current of the ionosphere[2,3], whereas the reconnection current on the magnetopause is almost entirely fed by the bow shock[4]. Guo et al.[5] presented a quantitative study of the bow shock contribution to the region 1 current, and concluded that more than 50% of the total region 1 current may originate from the
bow shock under strong southward IMF cases. Peng and Hu\textsuperscript{[6]} further investigated the bow shock contribution to the region 1 current under low Alfvén Mach numbers. Tang et al.\textsuperscript{[7]} demonstrated that the bow shock also contributes to the cross-tail current in the magnetosphere, and that such a contribution may exceed 80% of the total cross-tail current in certain situations. They also pointed out that as a result of the simultaneous feeding by the magnetopause and bow shock, the cross-tail current forms an overlapped $\theta$ structure instead of a single $\theta$ structure as admitted by assuming a single supply from the magnetopause. The studies mentioned above fully reflect the important role played by the bow shock in the electrodynamic coupling in the SMI system.

The bow shock current enters the magnetosphere via the magnetosheath. As an ideal MHD discontinuity, the bow shock must satisfy the Rankine-Hugoniot relations, from which the shock properties and surface current density may be evaluated from a given shape of the bow shock\textsuperscript{[8]}. From the current continuity equation, the divergence of the surface current density equals the normal volume current density, which flows into or out of the bow shock. The current flows from the magnetosheath to the bow shock when the divergence is positive, and in an opposite direction otherwise. To further understand the mutual relationship between the bow shock and the current in the magnetosphere and ionosphere, one needs to make a quantitative analysis of the distributions of the surface and normal volume current densities, which are closely related to the shape of the bow shock.

Empirical models were constructed for the shape of the bow shock in terms of statistical analyses of a vast amount of data of bow shock crossings by satellites, but most of them are based on the assumption of rotational symmetry of the shock with respect to the Sun-Earth line (the $x$ axis of the GSE coordinate system)\textsuperscript{[9,10]}. The presence of the Earth’s magnetic dipole field (the associated dipole moment is usually assumed to be in the direction opposite to the $z$ axis) and the appearance of the IMF will destroy the rotational symmetry of the SMI system and the bow shock. Even if the magnetopause is regarded as an obstacle of an ellipsoidal shape, which is rotationally symmetric with respect to the $x$ axis, so as to artificially exclude the influence of the Earth’s magnetic field, the shape of the bow shock obtained still depends on the orientation of the IMF. The bow shock possesses no any rotational symmetry with respect to the Sun-Earth line unless the IMF is exactly along the $x$ axis\textsuperscript{[11,12]}. It was found through statistical analyses of observations that the north-south width of the bow shock in the terminator plane ($x = 0$) is larger than the east-west width\textsuperscript{[13,14]}. For due southward IMF cases it was shown by MHD simulations\textsuperscript{[15]} that the geocentric distance of the intersection point of the bow shock with the $y$ axis is smaller than that with the $z$ axis when the IMF is weak ($B_{\text{IMF}_z} = -5, -10$ nT), which is consistent with observations described in Ref.[14]. The situation changes when the IMF is strong ($B_{\text{IMF}_z} = -15, -20$ nT), i.e., the north-south width of the bow shock in the terminator plane turns out to be smaller than the east-west width instead. These studies indicate that the bow shock is not rotationally symmetrical. The MHD simulation studies mentioned above replaced the magnetopause with an ellipsoid that is rotationally symmetric with respect to the Sun-Earth line\textsuperscript{[11,12]}, or confined themselves to generic southward IMF cases\textsuperscript{[15]}, and in addition, focused their attention on the configuration of the bow shock on the terminator plane or its sunward side.

Starting from MHD simulation data of the SMI system in simple cases with the IMF perpendicular to the Sun-Earth line, this paper investigates the geometry of the bow shock associated with different IMF strength $B_{\text{IMF}}$ and clock angle

$$\theta_{\text{IMF}} = \text{sign}(B_{\text{IMF}_y}) \arccos \left( \frac{B_{\text{IMF}_z}}{B_{\text{IMF}}} \right),$$  \hspace{1cm} (1)

where $B_{\text{IMF}_y}$ and $B_{\text{IMF}_z}$ are the IMF components in the GSE coordinate system ($B_{\text{IMF}_x} = 0$). According to Eq.(1), $\theta_{\text{IMF}}$ is the same as $B_{\text{IMF}_y}$ in sign, ranging from $-180^\circ$ to $180^\circ$, and the IMF is due northward for $\theta_{\text{IMF}} = 0^\circ$ and due southward for $\theta_{\text{IMF}} = \pm 180^\circ$. The paper differs from Refs.[11,12]: the magnetopause is determined consistently from global MHD simulations of the SMI system but not prescribed artificially as a rotational ellipsoid. The difference from Ref.[15] is that our analysis is aimed at various IMF clock angles without being limited to due southward IMF cases. Besides, we will examine the global configuration of the whole bow shock, and do not confine ourselves to that in the terminator plane and sunward region.
2 PHYSICAL MODEL AND DIAGNOSIS METHOD

The reader is referred to Refs.[16,17] for details of the physical models and simulation method. In what follows, we only give the physical parameters and describe the solution domain and numerical mesh.

The plasma density and thermal pressure are fixed to be 370 cm$^{-3}$ and 0.0465 nPa at the inner boundary of the magnetosphere ($r=3$ in the unit of the Earth's radius, taken as the unit of length hereinafter). The solar wind is along the Sun-Earth line with a velocity $v_{sw}=400$ km s$^{-1}$, number density $n_{sw}=5$ cm$^{-3}$, thermal pressure $p_{sw}=0.0126$ nPa, and the associated sonic Mach number is $M_s=8$. The magnetic dipole moment of the Earth points to the direction of negative $z$ axis. The IMF is perpendicular to the Sun-Earth line with adjustable strength $B_{\text{IMF}}$ and clock angle $\theta_{\text{IMF}}$: $B_{\text{IMF}}=5, 10, 20, 30$ nT; $\theta_{\text{IMF}}=0^\circ, 45^\circ, 90^\circ, 135^\circ, 180^\circ$. The corresponding Alfvén Mach numbers are $M_A=8.2, 4.1, 2.05, 1.37$ and the magnetosonic Mach numbers under perpendicular propagation regime are $M_{ms}=5.72, 3.65, 1.98, 1.35,$ respectively. The height-integrated conductances of the ionosphere are assumed to be uniform with a Pedersen conductance $\Sigma_P=5$ S and a Hall conductance $\Sigma_H=0$.

In the GSE coordinates $(x, y, z)$ the solution domain is $-300 \leq x \leq 30, -150 \leq y, z \leq 150$. It is discretized into a non-uniform mesh with 160×162×162 grid points, the minimum spacing being 0.4. For $B_{\text{IMF}}=30$ nT, the subsolar point of the bow shock has a geocentric distance larger than 30, so the solution domain is extended on its sunward side from $x=30$ to $x=50$ for this specific case, but the total number of grid points remains to be 160 along the $x$ direction. The ideal MHD equations in the solar wind-magnetosphere region are solved by the PPMLR-MHD scheme[16,17], whereas the ionospheric potential equation is solved by the successive over-relaxation iteration method.

After a quasi-steady numerical solution of the SMI system is obtained, we locate the bow shock by the following procedures: (1) Take a cylindrical coordinates $(R, \varphi, x)$ with its central axis to be the $x$ axis and its origin at the Earth center. $R$ denotes the perpendicular distance of the point of interest from the $x$ axis, and $\varphi$, the angle between the $x$-$y$ plane and the plane passing through the point of interest and the $x$ axis. A uniform mesh is laid out in the domain of $0 \leq R \leq 150, 0^\circ \leq \varphi \leq 360^\circ$ and $-300 \leq x \leq 30$, with spacings of $\Delta R=0.4, \Delta \varphi=1^\circ$, and $\Delta x=0.4$. (2) Map the simulation data from the GSE coordinates to the cylindrical coordinates via linear interpolation so as to obtain the discrete values of physical quantities at all grid points in the cylindrical coordinate system. (3) For a given set of grid coordinates $(R_i, \varphi_j)$, scan along the negative $x$ direction from the undisturbed solar wind region and locate the point $x_{ij}=f(R_i, \varphi_j)$, at which the ratio of the density to the background solar wind density equals 1.05. The function $x=f(R, \varphi)$ thus obtained determines the shape of the bow shock. A bow shock obtained by MHD simulations is spread over a transition layer of finite thickness. Here we have taken a small density ratio in order to ensure that the point found lies within the transition layer everywhere. To diagnose the cross section of the bow shock in the plane of $x=x_c$ ($x_c$ is a constant), we may confine the scan in the $x=x_c$ plane, and scan along the decreasing direction of $R$ for a given $\varphi_j$ starting from the undisturbed solar wind region, until the bow shock is met.

In order to examine the mutual relationship between the geometric configurations of the magnetopause and the bow shock, we need to roughly determine the configuration of the magnetopause based on the simulation data. There are four approaches: pressure gradient maximum, density gradient maximum, current density maximum, and streamline method. Different methods lead to nearly consistent results of the position of the magnetopause[18]. This paper uses the mass flux as the criterion and presumes that it equals one fourth of the background solar wind mass flux at the magnetopause. The magnetopause thus found is largely agreeable with the diagnosis result in terms of the density maximum method.

3 INFLUENCE OF THE EARTH’S MAGNETIC FIELD AND IMF ON THE GEOMETRY OF THE BOW SHOCK

Let us first make a qualitative analysis of the symmetry of the bow shock geometry before describing the simulation results. This paper has assumed that the Earth’s magnetic dipole moment points to the south, i.e.,
the negative $z$ direction, and the solar wind speed is along the Sun-Earth line, i.e., the negative $x$ direction. In this situation, the background solar wind parameters (velocity, density and thermal pressure) are all rotationally symmetric with respect to the Sun-Earth line, and the symmetry of the whole SMI system (including the bow shock) is completely determined by the symmetry inherent in the total strength of the compound magnetic field superposed by the Earth’s magnetic dipole field and the IMF. For the simple case, in which the IMF is perpendicular to the Sun-Earth line, the components of the compound magnetic field are expressed by

$$B_x = -\frac{3xz}{r^5}, \quad B_y = -\frac{3yz}{r^5} + B_{\text{IMF}y}, \quad B_z = \frac{x^2 + y^2 - 2z^2}{r^5} + B_{\text{IMF}z},$$  \hspace{1cm} (2)$$

where $r = (x^2 + y^2 + z^2)^{1/2}$ is the geocentric distance, and the magnetic field strength is normalized by that at the equator on the Earth’s surface. From Eq.(2) it can be immediately seen that the total strength of the compound magnetic field has a central symmetry with respect to the $x$ axis:

$$B(x, y, z) = B(x, -y, -z).$$  \hspace{1cm} (3)$$

Then the geometry of the bow shock has a central symmetry accordingly. The present paper is limited to this situation, so the numerical solutions of the bow shock obtained are all centrally symmetric with respect to the $x$ axis. The solution domain might be limited to the northern hemisphere because of the central symmetry, i.e., one half of the full solution domain mentioned above. Nevertheless, this paper has made numerical simulations in the full solution domain without invoking such a symmetry. In terms of the cylindrical coordinates $(R, \varphi, x)$, the central symmetry of the bow shock is expressed by

$$x(R, \varphi) = x(R, \varphi \pm 180^\circ).$$  \hspace{1cm} (4)$$

For cases with due southward or northward IMF ($B_{\text{IMF}y} = 0$), the compound magnetic field becomes stronger in symmetry, being symmetric with respect to the equatorial plane ($z = 0$) and the noon-midnight meridional plane ($y = 0$), namely,

$$B(x, y, z) = B(x, y, -z), \quad B(x, y, z) = B(x, -y, z),$$  \hspace{1cm} (5)$$

and the geometry of the bow shock becomes symmetric with respect to the two planes accordingly. Therefore, the solution domain might be limited to the northern-dusk quadrant, i.e., one fourth of the full solution domain mentioned above, for MHD simulations of the SMI system with due southward or northward IMF. Also, this paper does not take advantage of such symmetries, and has made simulations in the full solution domain. Note that Eq.(5) makes Eq.(3) satisfied automatically, i.e., the symmetry with respect to the equatorial and noon-midnight meridional planes must lead to a central symmetry with respect to the Sun-Earth line.

Incidentally, when $B_{\text{IMF}y}$ or $\theta_{\text{IMF}}$ is reversed in sign, the total strength $B_1$ before reversal and $B_2$ after reversal satisfy the following relation of symmetry

$$B_1(x, y, z) = B_2(x, y, -z), \quad B_1(x, y, z) = B_2(x, -y, z),$$  \hspace{1cm} (6)$$

as seen from Eq.(2). In other words, the total magnetic field strength is symmetric under spatial reflection with respect to the equatorial and noon-midnight meridional planes. Correspondingly, the configuration of the bow shock has the same symmetry under spatial reflection. Namely, if a bow shock associated with $B_1$ is obtained, then the bow shock associated with $B_2$ is simply a rotation by 180° around the $y$ or $z$ axis. Finally, once the IMF has an $x$ component, the compound magnetic field has no any symmetries mentioned above, so does the associated bow shock. The conclusions gained from the above simple analyses have been confirmed by MHD simulations in the whole solution domain made by us.

In what follows, we discuss the simulation results. In order to stress the influence of the Earth’s magnetic field on the shape of the bow shock, a simulation is made specifically for the case with $B_{\text{IMF}} = 0$. In this case,
the bow shock is symmetric with respect to the equatorial and noon-midnight meridional planes. However, owing to the action of the Earth's magnetic field, the SMI system as a whole, including the shape of the bow shock, has no rotational symmetry with respect to the Sun-Earth line. The results are shown in Fig. 1. Fig. 1a depicts the distribution of the logarithm of the number density (in the unit of cm\(^{-3}\)) in the equatorial \((x = 0)\) plane, the noon-midnight \((y = 0)\) plane, and the transverse section in the magnetotail \((x = -100)\) plane, along with the cross sections of the magnetopause and bow shock in the three planes. Fig. 1b gives the cross sections of the magnetopause (thin solid curves) and bow shock (thick solid curves) in the planes of \(x = 6, 0, -50\) and \(-100\), respectively, which stand more outside for smaller \(x\). In general, quasi-steady solutions of the SMI system resulting from MHD simulations oscillate slightly with time\(^{[16]}\), and thus the system somewhat deviates from its inherent symmetry. Nevertheless, such a deviation turns out to be not too large, so one can clearly see from Fig. 1b the symmetry of the magnetopause and bow shock with respect to the equatorial and noon-midnight meridional planes. Note that all cross sections of the magnetopause and bow shock are not in a circular shape, and thus have no rotational symmetry with respect to the Sun-Earth line. It is found by a careful examination of Fig. 1b that in the terminator plane (i.e., the dawn-dusk meridional plane, \(x = 0\)), the east-west width of the cross section of the bow shock is slightly larger than the north-south width, the ratio between the two widths is 1.011. While extending from the terminator plane toward the magnetotail, the bow shock cross section expands more rapidly along the north-south direction, forming a rhombus that is rounded in the four angles, wider from north to south and narrower from east to west. At \(x = -100\), the ratio between the east-west and north-south widths decreases to 0.922. The variation of the bow shock in shape is closely related the change of the shape of the magnetopause: the cross sections of the magnetopause are close to an ellipse, whose long half-axis transits gradually from east-west orientation on the sunward side to north-south orientation on the tailward side. The bow shock cross section in the \(x = 6\) plane on the sunward side (the innermost thick solid curve) has a ratio 1.092 between the east-west and north-south widths, and the corresponding magnetopause cross section (the innermost thin curve) has a similar shape, except that it is slightly concave along the north-south direction. Such a concavity is related to the shape of the magnetopause, which becomes concave in the polar cusp regions. The results above indicate that the main influence of the Earth's magnetic field is to cause the east-west width of the bow shock larger than the north-south width (about 9\%~11\%) nearby the terminator plane and on the sunward side, and smaller than the north-south width (by about 8\%) on the tailward side of the terminator plane. The magnetosphere presents a similar variation in shape, i.e., in the absence of the IMF the anisotropic interaction between the Earth’s magnetic field and the solar wind results in nearly synchronous, anisotropic

![Fig. 1 Results for B_{IMF}=0](image-url)  
(a) The distribution of \(\log(n)\) \((n\) is the number density in the unit of cm\(^{-3}\)) and the cross sections of the magnetopause and bow shock in the equatorial, noon-midnight meridian, and terminator planes.  
(b) The cross sections of the magnetopause and bow shock, denoted by thin and thick solid lines respectively in the planes of \(x = 6, 0, -50, -100\) and being more outside for smaller value of \(x\).
deformations of the magnetopause and the bow shock. This indicates that the Earth’s bow shock is rotationally
unsymmetrical even in simplest cases, where the solar wind velocity is perpendicular to the Sun-Earth line and
the IMF vanishes.

Let us now examine the influence of the IMF on the shape of the bow shock. Fig. 2 shows the numerical
results for cases with $\theta_{\text{IMF}}=0^\circ, 45^\circ, 90^\circ, 135^\circ, 180^\circ, \text{ and } -135^\circ$ ($B_{\text{IMF}}=10 \text{ nT}$). In each panel appointed for a
specific $\theta_{\text{IMF}}$, the cross sections of the bow shock (thick solid curves) and the magnetopause (thin solid curves)
are drawn for $x=6, 0, -50, \text{ and } -100$, respectively. Besides, in order to clearly see the influence of the IMF
on the shape of the bow shock, we have added the corresponding cross sections of the bow shock by dotted
curves for the case of $B_{\text{IMF}}=0$ in each panel of Fig. 2. Note that the magnetic field in the magnetosphere
obtained by global MHD simulations are closed for the due northward IMF case, and the closed magnetic field
lies on sunward side of the $x = -100$ plane. Thus the cross sections of the magnetopause are plotted only
in the $x = 6, 0, \text{ and } -50$ planes in Fig. 2a. As seen from Figs. 2a and 2e, both of the magnetopause and
the bow shock are symmetric with respect to the noon-midnight meridional and the equatorial planes for due
northward or southward IMF cases. Besides, the simulation results are given in Fig. 2f specifically for the case
of $\theta_{\text{IMF}} = -135^\circ$ in order to be compared with those for the case of $\theta_{\text{IMF}} = 135^\circ$. The two sets of results satisfy
the relation of symmetry under spatial reflection, given by Eq.(6).

Through a comparison of the bow shock cross sections in thick solid curves with their counterparts for the
case of $B_{\text{IMF}}=0$ in dotted curves, we may see that under the action of the IMF, the bow shock cross sections

![Fig. 2](image-url)
expand outward as a whole, and the extent of expansion increases when the IMF turns from north to south (i.e., $\theta_{IMF}$ increases). The effect of the IMF on the bow shock expansion has an apparent anisotropy: the expansion of the bow shock cross sections on the tailward side of the terminator plane is larger in extent along the direction perpendicular to the IMF than along the parallel direction, whereas the pattern of expansion of the cross sections on the sunward side of the terminator plane depends on $\theta_{IMF}$. For generic northward or duskward IMFs ($\theta_{IMF} = 0 ^\circ, 45 ^\circ, 90 ^\circ$), the expansion of the bow shock cross sections is also larger in extent along the direction perpendicular to the IMF than along the parallel direction, but the opposite holds for generic southward IMFs ($\theta_{IMF} = 180 ^\circ, 135 ^\circ$), i.e., the expansion extent is smaller along the perpendicular direction than along the parallel direction.

The position of the bow shock is determined mainly by two factors: the position of the magnetopause as the obstacle and the propagation speed of fast magnetosonic wave in the magnetosheath. The expansion of the magnetopause must lead to an expansion of the bow shock. The higher the fast magnetosonic wave speed is, the wider the width of the magnetosheath and thus the farther from the magnetopause the bow shock will be\cite{11,12}. In the presence of the IMF, it can be seen from a comparison between the bow shock cross sections shown in Fig. 2 and those shown in Fig. 1b for $B_{IMF} = 0$ that for the northward IMF case, the appearance of the IMF causes a global contraction of the magnetopause, which is stronger in extent along the direction perpendicular to the IMF than along the parallel direction. As a result, the magnetopause has an oblate shape, wide along the IMF and narrow along the perpendicular direction. When the IMF turns from north to south, the magnetopause remains to be in an oblate shape but gradually expands outward, and the width along the IMF direction is always larger than the width along the perpendicular direction. In terms of global MHD simulations, Hu et al.\cite{19} demonstrated that the magnetic reconnection rate of the magnetopause is roughly proportional to $\sin^{3/2}(\theta_{IMF}/2)$, and increases gradually as the IMF turns from north to south. Such a change of the magnetopause in shape is expected to be related to the magnetic reconnection on the magnetopause, which is not studied in the present paper. In the magnetosheath, the fast magnetosonic wave has a larger speed along the direction perpendicular to the magnetic field than that along the parallel direction. Consequently, the direction of stretch of the magnetopause is perpendicular to the maximum direction of the fast magnetosonic wave speed, and the effects of the two factors on the bow shock position happen to be opposite. The eventual shape of the bow shock depends on which effect dominates. It is the common influence of the magnetopause shape and the anisotropy of fast magnetosonic wave speed that results in a complicated variation of the bow shock cross sections in shape, as shown in Fig. 2.

Since the bow shock on the tailward side of the terminator plane is farther away from the magnetopause, the effect of the variation of the magnetopause in shape on it becomes weaker, and the effect of the anisotropy of fast magnetosonic wave speed always dominates. Therefore, the bow shock cross sections are narrow along the IMF and wide along the perpendicular direction. The shape of the bow shock cross sections on the terminator plane and its sunward side depends on $\theta_{IMF}$. For generic northward or duskward IMFs ($\theta_{IMF} = 0 ^\circ, 45 ^\circ, 90 ^\circ$), the magnetopause is close to the Earth, the stretch along the IMF is not remarkable, and thus the effect of the anisotropy of fast magnetosonic wave speed still plays a dominant role. Therefore, the bow shock on the sunward side is still narrower along the IMF than along the perpendicular direction. For generic southward IMFs ($\theta_{IMF} = 180 ^\circ, 135 ^\circ$), the expansion and stretch along the IMF rise and the effect of the shape of the magnetopause plays a dominant role, leading to a larger width of the bow shock cross sections along the IMF than along the perpendicular direction. Simulations were also made for other IMF strengths (5, 20, 30 nT), resulting in conclusions, which are essentially consistent with those made above for the case of $B_{IMF} = 10$ nT.

Incidentally, the orientation of the IMF affects the position of the subsolar point of the bow shock. For $\theta_{IMF} = 0 ^\circ, 45 ^\circ, 90 ^\circ, 135 ^\circ$, and $180 ^\circ$ ($B_{IMF} = 10$ nT), the geocentric distances of the subsolar point are found to be $x_0 = 14.4, 15.7, 15.8, 15.2$, and 14.3, respectively. In comparison with $x_0 = 14.3$ for the case $B_{IMF} = 0$, the positions of the subsolar point of the bow shock are close to each other, and the effect of the IMF orientation is within 10%. Simulations for other values of $B_{IMF}$ show that the results for $B_{IMF} < 10$ nT are close to those for
$B_{\text{IMF}} = 10 \text{ nT}$, i.e., the influence of $B_{\text{IMF}}$ and $\theta_{\text{IMF}}$ on $x_0$ is negligible. For cases with $B_{\text{IMF}} > 10 \text{ nT}$, however, an increase of $B_{\text{IMF}}$ leads to a decrease of $M_A$ and $M_{\text{ms}}$, and thus a rapid weakening of the bow shock and growing of $x_0$. For instance, for due southward IMF cases, when $B_{\text{IMF}}$ increases to 20 and 30 nT, $x_0$ reaches 19.2 and 30.7, respectively. Quite interestingly, the influence of the IMF orientation on $x$ scale parameters are denoted by $y_{\text{dusk}}$ and $z_{\text{north}}$, respectively, which are the characteristic scale parameters by taking the IMF as the datum direction. If one is essentially negligible. 19.2 and 30.7, respectively. Quite interestingly, the influence of the IMF orientation on $x$ scale parameters are denoted by $y_{\text{dusk}}$ and $z_{\text{north}}$, respectively. Actually, the shape of the bow shock presents a complicated variation along the Sun-Earth line, so its rotational asymmetry cannot be properly described only by parameters in the terminator plane. On the other hand, only for due northward or southward IMF cases can the bow shock be symmetric with respect to the equatorial and noon-midnight meridional planes, and only in this situation can $y_{\text{dusk}}$, $z_{\text{north}}$, and the ratio $y_{\text{dusk}}/z_{\text{north}}$ be used to characterize the rotational asymmetry. For the other cases, the above symmetries do not exist, so it is inadequate to use these parameters to characterize the geometrical properties of the bow shock cross sections in the terminator plane. Under this situation, it seems more reasonable to extract characteristic scale parameters of the bow shock cross sections according to the direction of the IMF. The procedures in determining such parameters are described below. When a cross section of the bow shock is available in a certain $x$ plane, two radial rays are drawn from the intersection point between the $x$ axis and the plane, one parallel and the other perpendicular to the IMF. The two radial rays intersect the bow shock cross section at a geocentric distance of $R_{b\parallel}$ and $R_{b\perp}$, respectively, which are the characteristic scale parameters by taking the IMF as the datum direction. If one takes the dawn-dusk ($y$ axis) and north-south ($z$ axis) directions as the datum directions, the corresponding scale parameters are denoted by $y_b$ and $z_b$. In the terminator plane, we have $y_b = y_{\text{dusk}}$ and $z_b = z_{\text{north}}$. Actually, Peredo et al.\cite{14} rotated the GSE coordinate system in the terminator plane so as to make the $z$ axis perpendicular to the IMF in the new coordinate system, and then redetermined the scale parameters $y_{\text{dusk}}$ and $z_{\text{north}}$. Obviously, the scale parameters thus determined happen to correspond to $R_{b\parallel}$ and $R_{b\perp}$ defined above. The difference lies only in that $y_{\text{dusk}}$ and $z_{\text{north}}$ defined by Peredo et al. is limited to the terminator plane, whereas $R_{b\parallel}$ and $R_{b\perp}$ are defined here for all $x$ planes, including the terminator plane.

From the simulation results for $B_{\text{IMF}} = 10 \text{ nT}$ and different values of $\theta_{\text{IMF}}$, we calculate the ratios of characteristic scale parameters, $y_b/z_b$ and $R_{b\parallel}/R_{b\perp}$, in three typical $x$ planes, which are shown in Table 1. The statistical averages listed in the table are obtained under the assumption that the IMF orientation has an isotropic distribution. It can be seen from the data of the ratio $R_{b\parallel}/R_{b\perp}$ that for all values of $\theta_{\text{IMF}}$, the bow shock cross sections on the tailward side ($x = -100$) have $R_{b\parallel}/R_{b\perp} < 1$, i.e., the width is smaller along the

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<th>Aspect ratio (\theta_{\text{IMF}}) (°)</th>
<th>$y_b/z_b$</th>
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<td>$x = 0$</td>
<td>$x = -100$</td>
</tr>
<tr>
<td>0</td>
<td>1.091</td>
<td>1.086</td>
</tr>
<tr>
<td>45</td>
<td>1.055</td>
<td>1.030</td>
</tr>
<tr>
<td>90</td>
<td>0.951</td>
<td>0.909</td>
</tr>
<tr>
<td>135</td>
<td>0.879</td>
<td>0.889</td>
</tr>
<tr>
<td>180</td>
<td>0.811</td>
<td>0.897</td>
</tr>
<tr>
<td>Average</td>
<td>0.965</td>
<td>0.954</td>
</tr>
</tbody>
</table>

For $\theta_{\text{IMF}} = 0°$ and $180°$, we have $y_b = R_{b\perp}$ and $z_b = R_{b\parallel}$, so that $y_b/z_b = (R_{b\parallel}/R_{b\perp})^{-1}$. For $\theta_{\text{IMF}} = 90°$, we have $y_b = R_{b\parallel}$ and $z_b = R_{b\perp}$, so that $y_b/z_b = R_{b\parallel}/R_{b\perp}$.\hfill

\textbf{Table 1 The aspect ratio of the cross section of the bow shock versus $\theta_{\text{IMF}}$ ($B_{\text{IMF}} = 10 \text{ nT}$) in various $x$ planes}.

For $\theta_{\text{IMF}} = 0°$ and $180°$, we have $y_b = R_{b\perp}$ and $z_b = R_{b\parallel}$, so that $y_b/z_b = (R_{b\parallel}/R_{b\perp})^{-1}$. For $\theta_{\text{IMF}} = 90°$, we have $y_b = R_{b\parallel}$ and $z_b = R_{b\perp}$, so that $y_b/z_b = R_{b\parallel}/R_{b\perp}$.
IMF than along the perpendicular direction. For generic northward or duskward IMF (θ_{IMF}=0°, 45°, 90°) cases, the bow shock cross sections on the terminator plane and its sunward side also have \( R_{b_y}/R_{b_\perp} < 1 \), i.e., the width is also smaller along the IMF than along the perpendicular direction. For generic southward IMF (θ_{IMF} = 180°, 135°) cases, the bow shock cross sections on the terminator plane and its sunward side have \( R_{b_y}/R_{b_\perp} > 1 \), i.e., the width is larger along the IMF than along the perpendicular direction. These results are in accordance with the qualitative conclusions drawn above from Fig. 2.

It will be difficult to make simple conclusions and physical explanations if one starts from the ratio \( y_b/z_b \). Taking the bow shock cross sections on the tailward side \((x = -100)\) of the terminator plane as examples, we have \( y_b/z_b > 1 \) for due northward or southward IMF cases, i.e., the east-west width of the bow shock cross sections is larger than the north-south width. For the other cases we have \( y_b/z_b < 1 \), i.e., the east-west width of the bow shock cross sections is smaller than the north-south width. The physical explanation of such conclusions cannot be made without considering the IMF orientation, since it has nothing to do with the east-west or north-south directions. Therefore, it is more reasonable to extract scale parameters by taking the IMF as the datum direction in characterizing the rotational asymmetry of the bow shock. Based on observations of bow shock crossings by spacecraft, Romanov et al.\(^{[13]}\) found that the north-south width of the bow shock is slightly larger than the east-west width. Peredo et al.\(^{[14]}\) presented a statistical analysis of 1392 crossings by 17 spacecraft to further confirm the above-mentioned conclusion, and pointed out that the north-south width of the bow shock in the terminator plane is larger than the east-west width by 2%~7%. From Table 1, in the terminator plane \((x = 0)\) \( y_b/z_b \) is larger than 1 for generic northward IMF (θ_{IMF}=0°, 45°) cases, but smaller than 1 for the other cases. Under the assumption of isotropic distribution of the IMF orientation, the average of \( y_b/z_b \) is 0.954, which agrees with the observational conclusion that the north-south width of the bow shock cross section in the terminator plane is larger than the east-west width. If analysis is based on the value of \( R_{b_y}/R_{b_\perp} \), then in the terminator plane \( R_{b_y}/R_{b_\perp} \) is larger than 1 for generic southward IMF (θ_{IMF}=180°, 135°) cases, but smaller than 1 for the other cases, and its statistical average is 0.961, i.e., the average statistical width is larger along the direction perpendicular to the IMF than along the parallel direction. Such a conclusion is also consistent with that made by Peredo et al.\(^{[14]}\) based on observations. When the bow shock on the tailward side \((x = -100)\) of the terminator plane is considered, the statistical average of \( y_b/z_b \) is 0.977, which is larger than the statistical average of 0.954 in the terminator plane. This would give a false impression that the bow shock cross sections on the tailward side seem to tend to be rotationally symmetrical. On the other hand, the statistical average of \( R_{b_y}/R_{b_\perp} \) is 0.858, which is smaller than the statistical average of 0.961 in the terminator plane. Now the conclusion is that the bow shock cross sections on the tailward side deviates farther from rotational symmetry. This is the case as seen from Fig.2, and indicates again the rationality of the use of scale parameters \( R_{b_y} \) and \( R_{b_\perp} \).

4 CONCLUSIONS

This paper analyzes the rotational asymmetry of the bow shock with respect to the Sun-Earth line in terms of global MHD simulations of the solar wind-magnetosphere-ionosphere (SMI) system. The simulations are limited to simple cases in which the solar wind speed is along and the Earth’s dipole moment and the interplanetary magnetic field (IMF) are perpendicular to the Sun-Earth line. In these cases, the quasi-steady SMI system and bow shock obtained are centrally symmetric with respect to the Sun-Earth line, and in particular, symmetric with respect to the equatorial and noon-midnight meridional planes for due northward and southward IMF cases.

However, even if the IMF strength vanishes, the bow shock has no rotational symmetry with respect to the Sun-Earth line. Under the influence of the Earth’s magnetic dipole field, the bow shock cross sections have an east-west width that is larger than the north-south width (by about 9%~12%) on the terminator plane \((x = 0)\) and its sunward side, and smaller than the north-west width (by about 8%) on the tailward side of the terminator plane. The shape of the cross sections are not an ellipse, but more like a rhombus that is rounded
in the four angles, wider from north to south and narrower from east to west. The magnetopause presents a similar variation in shape, i.e., in the absence the IMF, the anisotropic interaction between the Earth’s magnetic field and the solar wind leads to nearly synchronous, anisotropic deformations of the magnetopause and the bow shock.

In the presence of the IMF, the configuration of the bow shock is affected simultaneously by the shape of the magnetopause and the anisotropy of fast magnetosonic wave speed. The magnetopause expands outward, being stretched along the IMF, and the extent of its expansion and stretch increases with increasing IMF clock angle $\theta_{IMF}$ (i.e., when the IMF rotates from north to south). In the magnetosheath, the fast magnetosonic wave speed is higher in the direction perpendicular to the magnetic field than that in the parallel direction. Therefore, the stretch direction of the magnetopause is perpendicular to the maximum direction of the fast magnetosonic wave speed, and their effects on the bow shock position are exactly opposite. The eventual shape of the bow shock depends on which effect dominates. It is the common influence of the shape of the magnetopause and the anisotropy of fast magnetosonic wave speed that results in a complicated variation of the bow shock in shape. On the tailward side of the terminator plane, the anisotropy of fast magnetosonic wave speed dominates, so the cross section of the bow shock is wider in the direction perpendicular to the IMF. On the terminator plane and its sunward side, the shape of the bow shock cross section depends on $\theta_{IMF}$. The anisotropy of fast magnetosonic wave speed still dominates in generic northward or dawn-dusk IMF cases, so the corresponding bow shock cross section is still wider in the direction perpendicular to the IMF. Under generic southward IMF cases, on the other hand, the extent of the expansion and the stretch along the IMF of the magnetopause increases and the influence of the shape of the magnetopause dominates. Consequently, the corresponding bow shock cross section becomes narrower in the direction perpendicular to the IMF.

In light of the intimate relationship between the shape of the bow shock and the orientation of the IMF, we propose to take the IMF as the datum direction so as to extract the scale parameters of the cross sections of the bow shock in planes perpendicular to the Sun-Earth line, i.e., the geocentric distances of the intersection points of the bow shock cross sections along the directions parallel and perpendicular to the IMF, $R_{b//}$ and $R_{b\perp}$, respectively. These scale parameters provide a more reasonable description of the rotational asymmetry of the bow shock in comparison with the commonly used $y_b$ and $z_b$, which are defined as the geocentric distances of the intersection points of the bow shock cross sections along the dawn-dusk (the $y$ axis) and north-south (the $z$ axis) directions, respectively. Simulation data show that under the assumption of isotropic distribution of the IMF orientation, the statistical averages of $y_b/z_b$ and $R_{b//}/R_{b\perp}$ are both smaller than 1 in the terminator plane, which agrees with relevant observational conclusions\[14]\.

On the basis of the simulation results, the following issues should be taken into account in the simulation studies and observational analyses: (1) The assumption of rotational symmetry tacitly approved in existing empirical models of the bow shock\[9,10\] does not hold; one needs to consider the rotational asymmetry existing in reality in order to improve such empirical models. (2) Quadratic or elliptic curves were generally used to fit the observations in analyzing the bow shock cross sections in the terminator plane or other $x$ planes\[11,12,14\]. In fact, the cross sections of the bow shock are close to a rhombus rounded in the four angles or a parallelogram (see Fig. 2), which cannot be properly fitted by quadratic curves. It is necessary to probe better fitting methods, or to implement a three-dimensional fitting of the whole bow shock. (3) The commonly defined north-south and east-west widths of the bow shock cross section in the terminator plane cannot provide an adequate description of the geometry of the bow shock. It is more reasonable to use the geocentric distances along the IMF and its perpendicular direction. (4) In some MHD simulations, the magnetopause was replaced by an ellipsoidal obstacle of rotational symmetry\[11,12\], which completely excludes the influence of the shape of the magnetopause on the configuration of the bow shock. According to the present simulation results, such an influence is important on the bow shock configuration at least on the sunward side.
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REFERENCES