Theoretical properties of offset bipolar electric field solitary structures in space plasmas

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Abstract

The bipolar electric field solitary (EFS) structures have been frequently observed in the near Earth plasma regions, such as auroral zone, magnetopause, cusp regions, and magneto-tail. Sometimes these structures are observed as offset bipolar structures. In this paper, the properties of the offset bipolar EFS structures parallel to the magnetic field are studied with an ion fluid model in a cylindrical symmetry by considering electrostatic condition. The model results show that the offset bipolar EFS structures can develop from both ion-acoustic waves and ion cyclotron waves, and propagate along the magnetic field line in the space plasmas if plasma satisfies some conditions. The offset bipolar EFS structures can have both polarities. It will be first negative pulse and then positive pulse if the initial electric field $E_0 < 0$ or reverse in order if $E_0 > 0$. The amplitude of the offset bipolar EFS structures first decreases and then increases with the wave propagation velocity. Some results from our model are consistent with the observations.

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1. Introduction

The solitary waves play an important role in heating space plasma and accelerating charged particles (Mozer et al., 1980; Ergun et al., 2002; Drake et al., 2003). The electric field solitary (EFS) waves are characterized by bipolar structures (one negative peak and the other positive peak) parallel or perpendicular to the local magnetic field. These bipolar EFS structures are moving parallel to the background magnetic field and are found to be ubiquitous throughout the magnetosphere including the plasma sheet boundary (Matsumoto et al., 1994; Franz et al., 1998; Cattell et al., 1999), the upstream of the earth’s bow shock and magnetopause (Cattell et al., 2002; Behlke et al., 2004). The EFS waves were also observed in the auroral zone (Temerin et al., 1982; Boström et al., 1988; Mozer et al., 1997; Bounds et al., 1999; Cattell et al., 2001).

For the first time, Temerin et al. (1981) reported the S3-3 satellite observations from the auroral zone of paired ion electrostatic structures (offset bipolar structures) in which the separation between the bipolar peaks is much more than the characteristic width of each of the peaks. Pickett et al. (2008) observed the offset bipolar structures (positive part of the electric field pulse not following immediately the negative part or vice-versa) in the magnetosheath parallel to the magnetic field by Cluster satellite. Deng et al. (2006) reported the observations of offset bipolar structures parallel to the magnetic field from the reconnection at the diffusion region of day side magnetopause. Some of these structures have first negative pulse and then positive pulse or some have polarities in reverse order (Temerin et al.,...
1981; Pickett et al., 2008; Deng et al., 2006). Cattell et al. (2001) and Dombeck et al. (2001) reported that the amplitude of the EFS waves increases with the velocity.

Many studies have been done on the physical mechanism of the solitary wave. Most of the studies were restricted to the weakly nonlinear theories such as BGK (Bernstein–Greene–Kruskal) and the KdV (Kortweg–de Vries) equation. Washimi et al. (1966) first used a MHD model to study the ion solitary density waves and described them as rarefactive ion-acoustic solitary waves with KdV model to study the ion solitary density waves and described

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amplitude of the EFS waves increases with the velocity. Washimi et al. (1966) and Dombeck et al. (2001) reported that the amplitude ion-acoustic density solitons propagating obliquely to background magnetic field. Using the exact ion dynamics formulation Lee and Kan (1981) studied the nonlinear ion-acoustic density waves and solitons in a low-β magnetized plasma. Nonlinear periodic density waves in a cylindrically symmetric magnetic tube have been studied by Molotovshchikov and Ruderman (1987). Treumann et al. (1990) investigated a stationary solution for Alfven solitons propagating obliquely in a homogeneous magnetic field. Kalita and Bhatta (1994) investigated ion-acoustic solitary density waves in a warm magnetoplasma with electron drift. Witt and Lotko (1983) studied ion-acoustic solitary waves including offset bipolar EFS structure perpendicular to the magnetic field in a low-β magnetized plasma. For the offset bipolar EFS structure parallel to the magnetic field, no model has been reported, so far. Therefore, the physical mechanism of the offset bipolar EFS structure in the plasma is still need to be studied further.

In this paper, a fluid model in a low-β magnetized plasma parallel to the magnetic field in a cylindrical symmetry is proposed to study the offset bipolar EFS structures and their properties. In Section 2, we discuss the physical model. Numerical solution and analysis of the plasma conditions are discussed in Section 3. In the end, we present a brief summary and conclusion.

2. Physical model

We assume that the following conditions are satisfied in a tenuous space plasma. The fluid consists of electrons and ions, and plasma β ≪ 1, which implies that plasma pressure is much smaller than magnetic pressure. The wave scale λ is much larger than Debye radius, so charge separation effects can be neglected and the quasi-neutrality condition is satisfied, i.e., n ≈ n_e ≈ n_i (n is the particle number density). The magnetic field is constant and taken along the z-direction B_z. In consequence, we are merely looking for electrostatic solutions and the magnetic field will be passively taken into account in the gyrofrequency. Phase velocity satisfies v_T < v_p/γ < v_e, so we can neglect Landau damping. Here

v_T = (2T_0/m_e)^{1/2} is the particle thermal velocity, T_0 and m_e are particle energy and mass, subscript “r” and “e” denote the ions and the electrons, respectively, v_p is phase velocity, γ = cos θ and θ is the angle between the wave vector k and e_z. Because the electron mass is much smaller than the ion mass, we can neglect the electron inertia. To consider a cylindrical coordinate system, the fluid equations for the ions can be written as that described in detail in the paper by Shi et al. (2001), and

n = n_i ≈ n_e ≈ n_0 \exp(\epsilon/\phi_T) \quad (1)

where e is the elementary charge, \phi is the electric potential and n_0 is constant. In Eq. (1), quasi-neutrality is assumed and electron inertia is neglected which is valid for low frequency waves ω ≪ Ω_e.

From the fluid equations, we can get the linear dispersion relation as

ω^2(ω^2 - a c_s^2)(ω^2 - ω_0^2) = 0 \quad (2)

Here ω is the wave frequency, the coefficient a = T_e/Te, c_s = (T_e/m_e)^{1/2} is the ion-acoustic velocity, k is wave number, and Ω_e = eB/μ_0 c is ion gyrofrequency. From Eq. (2), we obtain that ω^2 = a c_s^2 and ω^2 = ω_0^2. These represent ion-acoustic wave and ion cyclotron wave, respectively. That means not only the ion-acoustic waves but also the ion cyclotron waves can be excited in the plasma described by our model and can, probably, develop into nonlinear waves.

For finding nonlinear solutions, we introduce the following dimensionless quantities: N = n/n_0, τ = Ω/t, R = r/ρ_i, Z = z/λ, Φ = eφ/T_e, M = v_p/c_s, v_p = ω/k (here ρ_i = c/Ω is the ion gyroradius), and the solution of the fluid equations that depend on r, θ, z, and t through the variable

S = (k_r r + k_z z - ω t)Ω_i/ω = (xR + γ z - τ M)/M \quad (3)

where x = sin θ, k_r and k_z are the components of k in the direction of r and z. Then from Eq. (1), we can write

N = \exp(Φ) \quad (4)

Thus, we get the electric field as

E = -1/N \cdot dN/dS \quad (5)

Considering \|k\|B, and assuming

V\|_{|S|} = 0, \quad N\|_{|S|} = 1 \quad (6)

and from the above equations, we obtain the energy integral of a classical particle in a 1-D “potential well” (Shi et al., 2001), as

1/2 \left( \frac{dN}{dS} \right)^2 + \psi(N) = 0 \quad (7)

Here,

\psi(N) = \left[ N \sqrt{1 - \frac{2}{\tau^2} \ln N} - 1 \right]^2 \left( \frac{N^2}{\tau^2} - 1 \right)^{1/2} \left( N \sqrt{1 - \frac{2}{\tau^2} \ln N} \right)^2 \quad (8)
According to Eqs. (5) and (7), we can obtain the electric field as
\[ E = -\frac{1}{N} \frac{dN}{dS} = \pm \sqrt{-\frac{2}{N^2} \psi(N)} \]  
(9)

Here, \( E_0 \) is the initial value of \( E \) and \( \psi(N) \) is the so-called “Sagdeev potential”. From Eqs. (7)–(9), we can analyze the nonlinear plasma density solution and electric field structure.

### 3. Numerical solution, analysis and discussion

From Eqs. (7)–(9), solutions for nonlinear waves can be obtained if the “Sagdeev potential” \( \psi(N) < 0 \). Our numerical solutions show that the waveform will be different if \( \psi(N) \) has different properties. Here, we choose \( S \) as the variable, that means the wave structure is investigated in the coordinate system moving together with the wave.

By analyzing the property of the “Sagdeev potential” \( \psi(N) \), we find that when the plasma parameters satisfy the condition below
\[ |(a/M^2 - 1)E_0| > 1, \quad a/M^2 > 1, \quad G_m \leq 1 + |(a/M^2 - 1)E_0| \]  
(10)

where
\[ G_m = \sqrt{\frac{a}{M^2}} \exp[(1 - a/M^2)/(2a/M^2)] \]  
(11)

there will be \( N_0 = \exp(M^2/(2a)) > 1 \), and \( \psi(N) \) has the properties: \( \psi(0) \rightarrow 0, \quad \psi(N_0) = 0, \quad \psi'(0) \rightarrow 0, \quad \psi'(N_0) > 1, \quad \psi(N) < 0 \) for \( 0 < N < N_0 \) (see Fig. 1). In Fig. 1, a typical Sagdeev potential is plotted as a function of \( N \) for Eq. (8) when \( E_0 = 0.2 \) and \( a/M^2 = 6.1 \). From Fig. 1, we can see that the \( \psi(N) \) has a maxima at \( N = 0 \) and a zero value with positive gradient at \( N = N_0 \), and \( \psi(N) \) is negative between these two points. So, Eq. (7) has a density soliton solution and Eq. (9) has solution of offset bipolar EFS structure corresponding to condition (10). Therefore, Eq. (10) is the condition for the existence of offset bipolar EFS waves.

Fig. 2 shows typical solitary structure and bipolar offset EFS waveforms for the condition (10) when \( E_0 = 0.2 \) and \( a/M^2 = 6.1 \). Fig. 2(a) shows an ion density hump soliton and the corresponding bipolar EFS structures are plotted in Fig. 2(b) and (c). Fig. 2(b) is plotted when the initial electric field \( E_0 < 0 \) which has first negative pulse and then positive pulse separated from each other. Fig. 2(c) is plotted when the initial electric field \( E_0 > 0 \) which has first positive pulse and then negative pulse separated from each other. From Eq. 2(a), we can see that the ion density soliton structure is broadened at the centre and have large gradients at the two shoulders. As a result, we can see in Fig. 2(b), the bipolar EFS waveform has sharp peaks which correspond to the two large gradients at the two shoulders of the ion density soliton structure and a relative flat region (with a very small electric field) between the two peaks which corresponds to the broadened top of the ion density soliton structure. The resultant offset bipolar EFS structure has first negative and then positive peak (see Fig. 2(b)). Fig. 2(c) is from the same calculation as that for Fig. 2(b) but with \( E_0 > 0 \). With a similar analysis to that for Fig. 2(b), we can see that the offset bipolar EFS structure becomes first positive and then negative (see Fig. 2(c)).

From Fig. 2, we can see that the bipolar EFS wave has amplitude of 2.65 and the duration of the bipolar EFS structure is about \( S = 8 \), the separation of the two peaks

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**Fig. 1.** “Sagdeev potential” \( \psi(N) \) plotted as a function of \( N \) for \( |E_0| = 0.2 \) and \( a/M^2 = 6.1 \). It has a maxima at \( N = 0 \) and increases when crossing \( N = N_0 \).

**Fig. 2.** Solitons and offset EFS waveforms for the condition (10) plotted as \( |E_0| = 0.2 \) and \( a/M^2 = 6.1 \). Panel (a) shows ion density hump soliton and the corresponding offset bipolar EFS waveforms are shown in panel (b) and panel (c) when the initial electric field \( E_0 < 0 \) and \( E_0 > 0 \), respectively.
the amplitude of the offset bipolar EFS structures first decreases and then increases with velocity. Cattell et al. (2001) and Dombeck et al. (2001) found that amplitude of the EFS waves increases with the velocity. From our model the amplitude of the offset bipolar EFS structure increases with phase velocity only when the Mach number $M > 0.39$.

4. Summary and conclusion

In this paper an electrostatic two fluid model for the offset bipolar EFS structures observed in space plasma is proposed. From the linear analysis, we concluded that the ion-acoustic waves and the ion cyclotron waves can be excited in the plasma described by our model and can, probably, develop into nonlinear waves. Sagdeev potential is then derived by considering the ion fluid equations for the parallel propagation in a low-$\beta$ space plasmas with cylindrical symmetry and bipolar offset EFS waveform is obtained from Eq. (9) for the condition (10).

The numerical results show that the offset EFS waves can develop not only from ion-acoustic waves but also from ion cyclotron waves when the plasma parameters obey the conditions $|aM^2 - 1|E > 1$, $aM^2 > 1$ and $G_m \leq 1 + |aM^2 - 1|E_0|$. The EFS structures can have first negative peak and then positive peak if the initial electric field is positive ($E_0 < 0$) or first positive peak and then negative peak if the initial electric field is negative ($E_0 > 0$) corresponding to the hump soliton. These conclusions are consistent with the observation. The results also show that the amplitude of the offset bipolar EFS structures does not monotonically vary with the wave velocity but first decreases and then increases with velocity. Therefore, offset bipolar EFS structures with both polarities (first negative peak and then positive peak or first positive peak and then negative peak) observed in space plasmas can be interpreted with our model.

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References


