Shock waves standing in the middle- and high-latitude magnetosheath from global MHD simulations

X. C. Guo,¹ C. Wang,¹ T. R. Sun,¹ and Y. Q. Hu²

Received 5 November 2010; revised 29 December 2010; accepted 11 January 2011; published 4 March 2011.

Standing shock waves (SSWs) are found to exist in the middle- and high-latitude magnetosheath through the global magnetohydrodynamic simulations. There are two (or one) SSWs for constant northward (or southward) interplanetary magnetic field (IMF); they extend into the magnetosheath region and further interact with the bow shock. Because of the extension of SSWs into the interplanetary space, especially when IMF turns southward, an indented bow shock emerges in front of the magnetosphere. The SSWs are excited by the indentations of the magnetopause in the supermagnetosonic solar wind flows in the magnetosheath; for northward IMF, one of the indentations is located in the cusp region and the other corresponds to the neutral point in the tailward of the cusp; for southward IMF, the indentation simply locates in the cusp region. We examine the Rankine-Hugoniot relations across the shock fronts and find the numerical model results are consistent with theoretical predictions.

1. Introduction

It is well known that a stationary collisionless hydromagnetic bow shock forms at about a few Earth radii in front of the magnetosphere, as a result of the interaction between the solar wind and the geomagnetic field. The shock wave decelerates the upstream supersonic solar wind plasma to be subsonic in the nose region; then the solar wind plasma continues to flow tailward around the sides of the magnetopause, it expands and becomes supersonic again at some point in the magnetosheath. Once any new obstacle is encountered by the supersonic plasma flow, naturally a second standing shock wave (SSW) would emerge in front of the obstacle, which is similar to the formation of the bow shock ahead of the magnetosphere. Walters [1966] was the first to predict the existence of this kind of SSW theoretically; he showed that the neutral points in the cusp would make indentations along the magnetopause surface in the noon-meridional plane, as well as the demarcation points (line) that separate field lines originating at low and high latitudes in other meridional planes, thus the bulging out of the magnetopause just behind the neutral points (demarcation line) presents a new obstacle to the supersonically flowing plasma. Therefore, a standing shock wave is expected to exist slightly upstream of the demarcation line bulge in the magnetosheath and be attached to the magnetosphere.

2. Simulation Model

2.1. Numerical Method

The Godunov’s method is a conservative numerical scheme for solving partial differential equations. The Godunov,
1959], and it can be used to solve the conservative ideal MHD equations numerically. Let us consider the one-dimensional partial equation with conservation form

$$\frac{\partial U}{\partial t} + \frac{\partial F(U)}{\partial x} = S(U),$$

here $U$ is the conservative variable (e.g., the density $\rho$, the momentum $\rho v$, the magnetic field $B$, and the total energy density $E$ in the ideal MHD equations), $F(U)$ is the corresponding flux, and $S(U)$ the nonhomogeneous source term. Let $x_{j+1/2}$ be the boundary between the $j$th and the $j \pm 1$st zones on the computational grid, we integrate the above equation over the zone with a size of $\Delta x_j = x_{j+1/2} - x_{j-1/2}$, and obtain the following discrete expression

$$U^n_{j+1} = U^n_j - \frac{\Delta t}{\Delta x_j} \left( F^n_{j+1/2} - F^n_{j-1/2} \right) + \Delta t S^n_{j+1/2},$$

here $\Delta t = t^n + t^{n+1}$, we define $U^n_j$ and $U^{n+1}_j$ as the average integration values of $U$ between $x_{j+1/2}$ and $x_{j-1/2}$ at time $t^n$ and $t^{n+1}$, and $S^n_{j+1/2}$ at time $t^{n+1}$. In order to obtain $U^n_{j+1}$ at time $t^n+1$, we should compute the flux $F^n_{j+1/2}$ defined as the local values at $x_{j+1/2}$ and $t^n + \Delta t/2$, and the source term $S^n_{j+1/2}$ at $t^{n+1}$. In order to obtain $F^n_{j+1/2} = F(U^n_{j+1/2})$, the values of $U^n_{j+1/2}$, also defined as the local values at $x_{j+1/2}$ and $t^n+1/2$, should be calculated for the time evolution of the conservation variable, and it is determined by the solution of the Riemann problem at the zone edge $x_{j+1/2}$. The two local states at the left and right sides for solving the Riemann problem are calculated from the variables at $t^n$, e.g., $U^n_j$ and $U^n_{j+1}$, depending on the interpolation functions chosen in the mesh. The original Godunov scheme treats the conservative variables as piecewise constant over the mesh zones at each time step, and it is first-order accurate in space. Van Leer [1979] extended the Godunov’s method in his Monotone Upstream-centered Schemes for Conservation Laws (MUSCL) scheme, and used piecewise linear approximations of each zone to replace the piecewise constant approximation of Godunov’s scheme, and resulted in a central difference scheme that is second-order accurate in space.

[7] The piecewise parabolic method (PPM) scheme of Colella and Woodward [1984] is a high-order extension of the MUSCL algorithm; the parabolae was introduced as the basic interpolation function in a zone to allow for a more accurate representation of smooth spatial gradients, and a steeper representation of captured discontinuities. It has a formal accuracy of third order in space and second order in time, with a low-numerical dissipation [Woodward and Colella, 1984]. Numerical results show that a shock structure can be captured within two numerical cells without any appreciable overshoot and undershoot using the PPM scheme.

2.2. Dissipation Mechanism

[8] For most of the simulation cases, the flattening procedure of the PPM scheme can eliminate the mild post shock oscillations; the small amplitude oscillation may occur for the case of the strong shocks. In some cases, it is necessary to introduce the numerical dissipation in the simulation code. As Colella and Woodward [1984] noted, one way of introducing dissipation is to perform the calculation on a constantly moving grid. If the zone edge is $x^n_{j+1/2}$ at time $t^n$, then

$$x^{n+1}_{j+1/2} = x^n_{j+1/2} + v^n_{j+1/2} \Delta t,$$

$$x^{n+1}_{j+1/2} = x^n_{j+1/2} - v^n_{j+1/2} \Delta t = x^n_{j+1/2},$$

here $v^n_{j+1/2}$ is a velocity which should be large only in the neighborhood of shocks, and has the form in one dimension

$$v^n_{j+1/2} = K \cdot \max(u_j - u_{j+1}, 0),$$

$u$ denotes the component velocity along $x$ direction; this treatment is equivalent to superimposing an advection velocity to the fluid motion which alternates direction every other time step. The obtained dissipation is that of the PPM advection scheme, and can be adjusted through the coefficient $K$. Note that the amount of dissipation required is quite small, e.g., $K = 0.1$ for PPM scheme, compared with what is typically used in convective finite difference schemes, e.g., $K = 1$ for MacCormack’s method for acceptable results, as shown by Woodward and Colella [1984]. In general, we adopt a simplified dissipation method based on the above content, the motion of the mesh grid has the form

$$x^{n+1}_{j+1/2} = x^n_{j+1/2} + K_0 \cdot \Delta x,$$

$$x^{n+1}_{j+1/2} = x^n_{j+1/2} - K_0 \cdot \Delta x = x^n_{j+1/2},$$

where $K_0$ is the adjusting coefficient, $\Delta x = \min(x_{j+1/2} - x_{j-1/2})$. If $K_0$ is nonzero, the convection effect is added in the global simulation domain, the dissipation operates regardless of the existence of shocks. In our simulation, $K_0$ is set to be zero for the northward IMF case, and 0.0001 for the southward IMF one.

2.3. Global MHD Simulation

[9] The PPMLR-MHD code [Hu et al., 2007] is a solar wind-magnetosphere-ionosphere coupling model, which is based on an extension of the Lagrangian version of the PPM. All the MHD-dependent physical conservative variables are defined at the zone centers as volume averages, and their spatial distributions are obtained by interpolation which is piecewise continuous, with a parabolic profile in each zone. Briefly, the algorithm of PPMLR-MHD consists of three steps:

[10] (1) A characteristic method similar to that proposed by Dai and Woodward [1995] is used to solve the Riemann problem at the zone edge $x_{j+1/2}$; we can obtain the local values of the conservative variables $U^n_{j+1/2}$ at the halftime point $t^{n+1/2}$ and further calculate the corresponding effective fluxes $F^n_{j+1/2}$.

[11] (2) Using the obtained fluxes $F^n_{j+1/2}$ and starting from the difference approximation of the Lagrangian conservation laws, we update all conservative variables from $t^n$ to $t^{n+1}$, the new conservative variables $U^{n+1}_{j+1}$ can be calculated in the Lagrangian coordinates.

[12] (3) The results $U^{n+1}_{j+1}$ are remapped back onto the fixed Eulerian grid through solving the corresponding advection equations.

[13] The ideal compressible MHD equations are solved using PPMLR-MHD algorithm in the GSM coordinate
system. We take a Cartesian coordinate system with the Earth center at the origin, the $x$, $y$, and $z$ axes point to the Sun, the dawn-dusk direction, and the north, respectively. The quarter simulation box extends from $x = 30R_E$ to $-300R_E$ along the Sun-Earth line and from 0 to $150R_E$ in the $y$ and $z$ directions, with a total of $450 \times 240 \times 240$ grid points. In the inner domain of $0 \leq |x|, y, z \leq 15R_E$, a uniform mesh is used with a mesh grid spacing of $0.1R_E$; the grid spacing increases according to a geometrical series of common ratio 1.05 along each axis. The interplanetary conditions can be adjusted through the front inflow boundary at $x = 30R_E$, the symmetric or asymmetric boundary conditions are set at $y, z = 0R_E$ depending on the physical parameters, and the other three outflow boundaries are set to be free. A magnetospheric-ionospheric electrostatic coupling model is imbedded in the inner boundary at $r = 3R_E$ to drive the inner magnetospheric convection. For simplicity, a uniform Pedersen conductivity of 5 Siemens is assumed, and no Hall terms are included in the ionosphere model.

3. Numerical Results

3.1. Initial State

Two cases with different IMF conditions are carried out in the simulation, including a due northward of 5 nT (case 1) and a due southward IMF of $-5$ nT (case 2). Other typical interplanetary conditions of the solar wind include a velocity of 400 km/s along the negative $x$ direction, a number density of $5 \text{ cm}^{-3}$ and a temperature of $0.91 \times 10^5$ K. All these physical parameters of the solar wind are input into the simulation box at the front boundary of $x = 30R_E$. The symmetric or asymmetric boundary conditions are set at $y, z = 0R_E$ depending on the physical parameters, and the other three outflow boundaries are set to be free. A magnetospheric-ionospheric electrostatic coupling model is imbedded in the inner boundary at $r = 3R_E$ to drive the inner magnetospheric convection. For simplicity, a uniform Pedersen conductivity of 5 Siemens is assumed, and no Hall terms are included in the ionosphere model.

3.2. Standing Shock Waves

After about one physical hour for the nonlinear evolution of the magnetosphere from the above initial state, the simulation result reaches a saturation state. Figure 1 shows the color contours of the $x$ component velocity ($V_x$) in the noon-meridional plane at $t = 6000 \tau_A$ ($\tau_A$ is a normalized unit, equal to be 0.935 seconds). The solid lines indicate the magnetic field lines originating from the Earth (blue) and the interplanetary space (red). For the case of northward IMF, we can clearly identify the magnetopause boundary that sandwiches between the open (red) and closed (blue) field lines. The centered sphere, with a radius of 6 $R_E$, covers the regions near the inner magnetospheric boundary to highlight the rest of the simulation domain.

The right arrow points to the approximate location of the throat of the cusp, which is at a latitude of about $70^\circ$ for case 1 and about $55^\circ$ for case 2; the latitude of the cusp moves to a lower latitude when IMF turns from northward to southward; the results agree with a latitude range from about $70^\circ$ to about $45^\circ$ that is based on observations [Farrell and Van Allen, 1990], as well as previous simulation works by Siscoe et al. [2005]. As Figure 1a shows, the left arrow points to the neutral point that resides about 5 $R_E$ tailward of the cusp; the merging antiparallel magnetic field lines and the high-speed plasma flows with inverse directions near the neutral point indicates the occurrence of magnetic reconnection. This kind of magnetic reconnection is referred to as lobe reconnection that occurs when IMF turns northward.
shock, SSW1 and SSW2 in order. Because of the interaction between the bow shock and the two SSWs, the high-latitude bow shock is not as smooth as that at the lower latitudes where the SSWs are not taken into account.

[19] For case 2, shown in Figure 1b, the cusp has a lobe shoulder that presents a much larger obstacle than that of northward IMF cases; consequently the SSW appears much larger and interacts with the bow shock, which leads to the formation of a distinct indented bow shock. The label of “SSW3” indicates the presence of SSW corresponding to the indented cusp region; unlike case 1, there are no other SSWs for the case of southward IMF because of the absence of lobe reconnection at the tailward magnetopause of the cusp.

[20] The formation of the SSWs can be explained by the prediction of Walters [1966]: the bow shock reduces the supersonic solar wind plasma to be subsonic near the nose region, but as the plasma continues its journey tailward around the sides of the magnetosphere, it expands and becomes supersonic again at some point in the magnetosheath; there are two indentations for case 1 (the cusp throat and the neutral point), and one indentation for case 2 (the cusp throat), being relative to the other smooth surface of the magnetopause. The tailward parts of the magnetopause behind the indentations are obstacles for the supersonic flows, then the SSWs are required to reduce the supersonic solar wind plasma to be subsonic to flow around the new obstacles.

[21] The SSWs appear not only in the noon-meridional plane, but also in the other meridional planes. Figure 2 presents the color contours of a half northern hemisphere surface with a fixed radius of 16 \( R_E \) (Figure 2a, case 1) or 12 \( R_E \) (Figure 2b, case 2), the background color is valued by the \( x \) component velocity; the surface region with latitudes lower than 45° (Figure 2a) or 30° (Figure 2b) are not shown. In Figure 2a, two obvious and sharp discontinuities, the bow shock and the magnetopause, are distinguished. The two SSWs are weaker than the bow shock, sandwiching between the bow shock and the magnetopause; they extend to a lower latitude of about 70°. Modulated by the geometrical indentations on the magnetopause surface, the extension of the SSWs into the flank is related to the width of the cusp, and the separator line of the lobe reconnection predictively. In Figure 2b, only the magnetopause and the SSW can be distinguished; the SSW3 mainly locates at latitudes ranging from 40°–50°. We also examine the distributions of the SSWs with a various sampling sphere with different radius, the results indicate that the SSWs exist mainly with a latitude higher than 70° for case 1, and latitudes ranging from 40°–50° for case 2. Lower latitudes of the cusp region correspond to lower latitudes of the SSWs.

3.3. Rankine-Hugoniot Relations

[22] Although the theoretical prediction [Walters, 1966] and the simulations have indicated the existence of the SSWs, it is necessary to examine the Rankine-Hugoniot (R–H) relations between the two states across the shock fronts. As Figure 3 shows, we choose a white solid line segment across the SSWs. For case 1, in the \( x-z \) plane, they are set from (3.3, 12.8)\( R_E \) to (5.3, 12.8)\( R_E \) for L1 in Figure 3a, and from (0.0, 14.0)\( R_E \) to (2.0, 14.0)\( R_E \) for L2 in Figure 3b. For case 2, the line segment is set from (7.0, 10.0)\( R_E \) to (9.0, 10.0)\( R_E \) for L3 in Figure 3c. Shock front segment
locations are indicated with white dotted lines; furthermore, the line segments L1, L2 and L3 are approximately perpendicular to the shock fronts of the SSWs. Corresponding to the cusp throat and the neutral point, the indentations are clearly shown through the color contours of $V_x$ in Figure 3.

Figure 4 shows the profiles of the number density $N$, the $x$ component velocity $V_x$, the $z$ component magnetic field $B_z$, and the total energy density $U (\text{nJ/m}^3)$ along the three line segments. The nearly in-phase relation between $N$ and $|B_z|$ indicates that the SSWs are all fast standing shock-like waves. In each panel, the two dotted red lines plot the approximate locations of the upstream and downstream of the shock waves, then we display the corresponding physical parameters in Table 1 (Upst and Down). As Figure 3 shows, the interplanetary magnetic field lines intersect the three shock fronts obliquely in the magnetosheath, thus the SSWs are standing oblique shocks. We can calculate the normal fast magnetosonic and normal
Guo et al.: Shock Waves in the Magnetosheath

Table 1. Shock Parameters of the Three SSWs and the Comparisons Between the Simulated (Down) and Theoretical (Down(T)) Values of the Downstream

<table>
<thead>
<tr>
<th></th>
<th>N(cm⁻³)</th>
<th>Vₓ(km/s)</th>
<th>Bₓ(nT)</th>
<th>U(nJ/m⁴)</th>
<th>Mₓ</th>
<th>Mᵧ</th>
</tr>
</thead>
<tbody>
<tr>
<td>SSW1</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Upst</td>
<td>11.9</td>
<td>-218</td>
<td>12.7</td>
<td>1.22</td>
<td>1.15</td>
<td>2.18</td>
</tr>
<tr>
<td>Down</td>
<td>13.4</td>
<td>-195</td>
<td>14.4</td>
<td>1.32</td>
<td>0.99</td>
<td>1.88</td>
</tr>
<tr>
<td>Down(T)</td>
<td>14.5</td>
<td>-180</td>
<td>15.6</td>
<td>1.41</td>
<td>0.88</td>
<td>1.72</td>
</tr>
<tr>
<td>Error</td>
<td>0.08</td>
<td>0.08</td>
<td>0.08</td>
<td>0.06</td>
<td>0.12</td>
<td>0.09</td>
</tr>
<tr>
<td>SSW2</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Upst</td>
<td>8.77</td>
<td>-234</td>
<td>10.1</td>
<td>0.911</td>
<td>1.29</td>
<td>2.08</td>
</tr>
<tr>
<td>Down</td>
<td>11.4</td>
<td>-180</td>
<td>14.4</td>
<td>1.07</td>
<td>0.90</td>
<td>1.47</td>
</tr>
<tr>
<td>Down(T)</td>
<td>12.4</td>
<td>-166</td>
<td>15.2</td>
<td>1.18</td>
<td>0.80</td>
<td>1.40</td>
</tr>
<tr>
<td>Error</td>
<td>0.08</td>
<td>0.08</td>
<td>0.05</td>
<td>0.09</td>
<td>0.12</td>
<td>0.05</td>
</tr>
<tr>
<td>SSW3</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Upst</td>
<td>14.5</td>
<td>-210</td>
<td>-12.6</td>
<td>1.51</td>
<td>1.08</td>
<td>2.48</td>
</tr>
<tr>
<td>Down</td>
<td>17.8</td>
<td>-161</td>
<td>-15.6</td>
<td>1.77</td>
<td>0.76</td>
<td>1.79</td>
</tr>
<tr>
<td>Down(T)</td>
<td>16.3</td>
<td>-187</td>
<td>-14.2</td>
<td>1.65</td>
<td>0.93</td>
<td>2.14</td>
</tr>
<tr>
<td>Error</td>
<td>0.09</td>
<td>0.14</td>
<td>0.10</td>
<td>0.07</td>
<td>0.18</td>
<td>0.12</td>
</tr>
</tbody>
</table>

Alfvén Mach numbers through the relations $M_f = V_f/v_f$ and $M_A = V_A/v_A$, where $V_f$ corresponds to the fast magnetosonic speed

$$V_f^2 = \frac{1}{2} (V_A^2 + C_S^2) + \frac{1}{2} \left( V_A^2 + C_S^2 - 4V_A^2C_S^2 \cos^2 \theta \right)^{1/2}. $$

[24] $\theta$ is the angle between the shock normal and the magnetic field, $V_A$ and $C_S$ are the Alfvén and acoustic speeds, respectively. The SSWs are standing in the magnetosheath, which indicates that the shock speed is zero relative to the GSM coordinate system; the $R$-$H$ relations can be expressed as a conservation form as follows:

$$[\rho V_x] = 0,$$

$$[\rho V_x^2 + P + B^2/2\mu_0] = 0,$$

$$[\rho V_x V_z - \frac{1}{\mu_0} B_x B_z] = 0,$$

$$[\rho V_z (V_x^2 + \frac{\gamma P}{\mu_0} V_z + \frac{B_x^2}{\mu_0} V_z) - B_x V_z + \frac{B_x B_z}{\mu_0}] = 0,$$

$$[B_x] = 0.$$

[25] According to the above relations, once the upstream is set, the theoretical parameters of the downstream can be obtained. The theoretical results of the downstream are shown as Down(T) in Table 1 after numerical calculation, here the number density $N = \rho/m_p$, $m_p$ is the mass of a proton; $U = \rho U^2/2 + P(\gamma - 1) + B^2/(2\mu_0)$, is the total energy density.

[26] We can see that the supermagnetosonic solar wind plasma becomes submagnetosonic over the three shock fronts, the fast magnetosonic Mach number changes from 1.15 to 0.99 for the SSW1, from 1.29 to 0.90 for the SSW2, and from 1.08 to 0.76 for the SSW3; the Alfvén Mach number is larger than 1 over the three shock fronts. The transitions of the Mach numbers indicate that they are all fast shock waves. The simulated values (Down) are nearly consistent with the theoretical results (Down(T)). For the SSW1, the relative error between the theoretical and simulation results are equal to or less than 0.09, except for that of the fast magnetosonic Mach number (0.12). The SSW2 and the SSW3 have similar characteristics as the SSW1, which can be seen from Table 1. Note that the error is acceptable, it is mainly because the two states of the SSW are not completely uniform in Figure 4.

[27] Fore case 1, the shocked submagnetosonic solar wind plasma that has passed through SSW1 can be accelerated, and become supermagnetosonic again in front of the SSW2. The distance between SSW1 and SSW2 is about 3.4 $R_E$ along the $x$ direction. Although the two shocks are relatively weak compared with the bow shock, we can conclude that they do persist in the high-latitude magnetosheath during our simulation.

4. Discussion and Summary

[28] In this paper, we have simulated two SSWs in the magnetosheath during a period of northward IMF $B_z = 5$ nT. This result somewhat differs from the one SSW concept of the theoretical prediction. This discrepancy is caused by the difference between the theoretical Chapman and Ferraro [1940] model and the MHD model. The Chapman-Ferraro model is a vacuum model, the neutral points mark the boundary in the noon-meridional plane between low-latitude geomagnetic field lines that close on the dayside of the magnetosphere, and high-latitude field lines that are swept back to form the magnetospheric tail; the neutral point is within the cusp. Further, the cusp and the neutral point are treated as the same indentation on the magnetopause in the theoretical analysis. Thus, there is only one SSW existing in the northern (southern) magnetosheath.

[29] The MHD model is based on the fluid description of the plasma. The cusp refers to the region where magnetospheric field lines converge at the Earth, as Figure 1 shows; when IMF turns northward, part of the dayside magnetospheric field lines stretch to the tailside of the cusp; the cusp is not exposed to the solar wind plasma directly, because the solar wind plasma cannot penetrate the magnetic field lines perpendicularly and reaches the cusp for the presence of the “frozen-in” condition. These results have been mentioned by previous researchers [e.g., Wu, 1983]. A bulging out of the magnetopause appear behind the cusp; then the first SSW appears. The neutral point is shifted to the tail side of the magnetopause, about 5 $R_E$ away from the cusppoint. Lobe magnetic reconnection occurs near the neutral point, which leads to a second geometrical indentation along the magnetopause in the noon-meridional plane. A new bulging out of the magnetopause appears just behind the neutral point, which leads to the formation of the second SSW in our simulation.

[30] For the case of southward IMF, there is only one indentation on the surface of the magnetopause because of the absence of the lobe reconnection. The picture of one SSW coincides with the theoretical prediction by Walters [1966]. Moreover, the SSW is much stronger than those of northward IMF; because of the interaction between the bow shock and the SSW, there is a indentation on the surface of the northern (or southern) bow shock. It seems that
the indented bow shock has never been mentioned in previous studies describing the geometries of bow shock modeling [e.g., Formisano, 1979; Merka and Szabo, 2004], it is directly caused by the indented magnetopause from the viewpoints of gas dynamics.

[31] We also tested cases with different IMF strengths and solar wind speeds, and obtained similar results except for variations of the latitudes of SSWs and the shock strength. Comparing with our previous works [e.g., Hu et al., 2007], we have increased the resolution of the numerical mesh grids to a higher level of 0.1 R_E. This treatment helps us to observe more physical details than in our previous work. At the same time, the numerical scheme of PPM provides a powerful method to capture the weak shocks, e.g., the SSWs in our simulation. As we have tested, the SSWs are sensitive to the numerical dissipation, they disappear once certain numerical dissipations are added into the simulation code, e.g., $\alpha = 0.1$ in our cases. That is probably why few cases of the SSWs have been found in previous simulation work, because a relatively larger numerical dissipation may exist for coarser mesh grids or relatively simpler numerical schemes.

[32] Toth [2000] showed that in some circumstances it is possible to get the wrong jump conditions across MHD shocks if the divergence free constraint ($\nabla \cdot B = 0$) is not satisfied. As we have shown, the R-H relations across the shock fronts are well examined, and the relative error between the simulated and theoretical values are acceptable; all these facts confirm the physical existence of SSWs in the magnetosheath in our simulation.

[33] Note that there is still no observational evidence to confirm the existence of SSWs near the cusp and the neutral point, although the theoretical and simulation results have predicted their existence based on the fluid model. Thus, finding direct evidence of the SSWs at the high-middle-latitude magnetosheath is a challenge for future observational research.

[34] Acknowledgments. This work was supported by NNSFC grants 40804044, 40921063, and 40831060 and in part by the Specialized Research Fund for State Key Laboratories of China.

[35] Masaki Fujimoto thanks the reviewers for their assistance in evaluating this paper.

References