Motion of observed structures calculated from multi-point magnetic field measurements: Application to Cluster

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A new method is described which calculates the velocity of observed, quasi-stationary structures at every moment in time from multi-point magnetic field measurements. Once the magnetic gradient tensor \( G = \nabla \vec{B} \) and the time variation of the magnetic field have been estimated at every moment, the velocity can then be determined, in principle, as a function of time. One striking property of this method is that we can calculate the velocity of structures for any dimensionality: for three-dimensional structures, all three components of the velocity vector can be calculated directly; for two-dimensional (or one-dimensional) structures, we can calculate the velocity along two (or one) directions. The advantage of this method is that the velocity is determined instantaneously, point by point through any structure, and so we can see the time variation of the velocity as the spacecraft traverse the structure. In this paper, the feasibility of the method is tested by calculating the motion velocity of a three-dimensional, near cusp structure and a two-dimensional magnetotail current sheet. The results for one-dimensional structures in the magnetopause and cusp boundaries are compared to calculations for the standard techniques for analyzing discontinuities. Citation: Shi, Q. Q., C. Shen, M. W. Dunlop, Z. Y. Pu, Q.-G. Zong, Z.-X. Liu, E. A. Lucek, and A. Balogh (2006), Motion of observed structures calculated from multi-point magnetic field measurements: Application to Cluster, Geophys. Res. Lett., 33, L08109, doi:10.1029/2005GL025073.

1. Introduction

The motion velocity of various observed structures, such as Flux Tube, Bow Shock, magnetopause, and magnetotail current sheet have been widely studied by analyzing the observed parameters of one single satellite or multi-satellite using different tools. Examples of techniques using one single satellite are DeHoffmann-Teller analysis (HT) [see Paschmann and Daly, 1998, hereinafter referred to as PD, chapter 9], minimum Faraday residue analysis (MFR) [Khrabrov and Sonnerup, 1998], and minimum mass flux residue analysis (MMR) [Sonnerup et al., 2004]. Examples of multi-satellite, which have been applied to 1-D bound-aries, techniques are: the timing method (called constant velocity approach (CVA) by Haaland et al. [2004]) [see Russell et al., 1983; PD, chapters 10, 11, 12, and 14], Discontinuity Analysis (DA) method [PD, chapter 11], the timing method of constant thickness approach (CTA) and derivatives [Haaland et al., 2004 and the reference therein], the MVAJ method [Haaland et al., 2004], the curvature estimating method [Shen et al., 2003], and so on. Most of the above methods except DA and CTA assume a constant velocity or constant acceleration, and all the above multi-point techniques apply the calculation only to one-dimensional structures. In the present paper, we briefly describe, and test by the use of Cluster data, a method for calculating the velocity for three-dimensional (3-D), two-dimensional (2-D) or one-dimensional (1-D) quasi-stationary structures at every observed moment in time using multi-point magnetic field data and knowledge of the relative positions of the spacecraft. The dimensionality of each observation is not assumed, but can be determined using the “Minimum Directional Derivative” (MDD) technique [Shi et al., 2005]. The velocity of structures in the near cusp region, magnetopause, and the magnetotail current sheet is then calculated.

2. Method

As we know, the total time derivative of the magnetic field observed by the spacecraft is \( d\vec{B}/dt = (\partial\vec{B}/\partial t)_\text{str} - \vec{V}_\text{str} \cdot \nabla \vec{B} \), where the first term on the right is the temporal variation in the reference frame of the structure. The second term on the right is the temporal variation caused by the motion through spatial gradients [see Song and Russell, 1999], and \( \vec{V}_\text{str} \) is the velocity of the structure relative to the observer, that is, the spacecraft.

The basic assumption of this method is that the structure to be analyzed does not change its configuration significantly during the interval when the satellite system moves across it. That is, it is a quasi-stationary structure. So, in the reference frame of the structure, \( (\partial\vec{B}/\partial t)_\text{str} \sim 0 \). We get

\[
d\vec{B}/dt + \vec{V}_\text{str} \cdot \nabla \vec{B} = 0. \tag{1}
\]

Equation (1) means that the observed temporal change of the magnetic field by the spacecraft is only caused by the motion of the structure. The main idea of this method is to solve the difference equations of Equation (1) at every observed point: \( d\vec{B}/dt \) can be estimated by calculating the magnetic field time difference observed by the spacecraft at some time series resolution; matrix \( \nabla B \) can be estimated by many multi-point methods [see the references in Shi et al., 2005]. For a 3-D structure, the three components of \( \vec{V}_\text{str} \) can
be directly calculated by solving the difference equations of Equation (1), expanded as three linear equations with three unknowns.

[5] For 1-D or 2-D structures, there must be at least one direction \( \hat{n} \) satisfying \( \partial/\partial n \sim 0 \) and then \( \text{det}(\nabla \hat{B}) \sim 0 \), so that directly solving the Equation (1) will produce inaccurate solutions. We therefore expect that from the magnetic field data, we can only obtain the velocity along one direction for a 1-D structure, and along two directions for a 2-D structure. To solve this problem, we have developed a dimensional analysis method called “Minimum Directional Derivative” (MDD) analysis [Shi et al., 2005] to find a structure’s dimensionality and its characteristic (principal) directions, using multi-point magnetic field measurements. It is briefly described here: first estimate the magnetic gradient tensor \( G = \nabla \hat{B} \) at every moment by multi-point measurements, and second find the eigenvalues and eigenvectors of a symmetrical matrix \( L = GG^T = (\nabla \hat{B})(\nabla \hat{B})^T \) (The superscript \( T \) denotes transposition). The three eigenvalues \( \lambda_1, \lambda_2 \) and \( \lambda_3 \) represent the maximum, intermediate and minimum values of the magnetic field directional derivatives, and the three eigenvectors \( \hat{n}_1, \hat{n}_2 \) and \( \hat{n}_3 \) represent the corresponding directions. We say that: (1) if \( \lambda_1, \lambda_2 \) and \( \lambda_3 \) are not quite different from each other, we can regard it as a 3-D structure; (2) if \( \lambda_1, \lambda_2 \gg \lambda_3 \), we can regard it as a quasi-2-D structure and its invariant direction is along \( \hat{n}_3 \), i.e., \( \partial/\partial \hat{n}_3 = 0 \); (3) if \( \lambda_1 \gg \lambda_2, \lambda_3 \), we regard it to be a quasi-1-D structure, with the invariant axes in the plane of \( \hat{n}_2 \) and \( \hat{n}_3 \) and the variant direction along \( \hat{n}_1 \).

[6] Once the structure’s dimensionality and the three principal directions are determined, we find that 1-D and 2-D problems can be solved in either of the following two ways. One way is to solve the problem in the MDD eigenvector-based coordinate system. Equation (1) can be written as \( \frac{d\hat{B}}{dt}(\nabla \hat{B})^T = -\vec{V}_{str} \cdot (\nabla \hat{B})(\nabla \hat{B})^T \), where \( T = (\hat{n}_1, \hat{n}_2, \hat{n}_3) \) is the transformation matrix from the original (e.g., GSE) to the MDD eigenvector-based coordinate system. We get

\[
\frac{d\hat{B}}{dt}(\nabla \hat{B})^T = -(\vec{V}_{str})_{MDD} \cdot \Lambda
\]

where from the paper of Shi et al. [2005], we know that \( \Lambda = T^T(\nabla \hat{B})(\nabla \hat{B})^T T \) is a diagonalized matrix, of which the diagonal terms are the three eigenvalues \( \lambda_1, \lambda_2, \lambda_3 \) of the \( L \) matrix introduced above. And \( (\vec{V}_{str})_{MDD} = \vec{V}_{str} \cdot T \) is the velocity in the eigenvector coordinate system. Then we can solve the difference equations in (2) one by one if the corresponding eigenvalue is significant. That is, for a 1-D structure, we just solve the first equation corresponded to the largest eigenvalue \( \lambda_1 \) and then get the velocity along its direction of variation, i.e., its normal; while for a 2-D structure, we only solve the first two equations related to \( \lambda_1 \) and \( \lambda_2 \) and obtain the velocity components along the maximum and intermediate derivative directions. Another way is to first calculate the three components of the velocity by solving the difference equations of Equation (1), and then project the result to the three directions \( (\hat{n}_1, \hat{n}_2, \hat{n}_3) \). The velocities along the maximum direction (for a 1-D structure) or maximum and intermediate directions (for a 2-D structure) can have a relatively reliable accuracy, but the other direction(s) will not. Since generally the variations are not exactly 1-D or 2-D and the corresponding eigenvalues are not exactly zero (then the \( \text{det}(\nabla \hat{B}) \) is not strictly equal to zero), The above two solutions are intrinsically identical in the numerical calculation: the transformation to the MDD coordinate system before (i.e., in the 1st way) or after (i.e., in the 2nd way) the calculation can give the same results. In practice, we can use both of them and cross-check each other. We call our method of velocity calculation for any dimensionality: the “Spatio-temporal Difference” (STD) on the magnetic field.

[7] One type of the error in the STD method comes from our stationary assumption, i.e., \( \langle \partial \hat{B}/\partial n \rangle_{str} \sim 0 \), which means we can calculate its velocity only if the structure itself changes very slowly on the time scale of the motion. Other sources of error exist in the difference equations of Equation (1), including the measurement error of the magnetic field, the error in the determination of the satellite relative position, and the truncation errors caused by the approximate difference of the two terms in Equation (1). Generally, the truncation errors dominate, which give us two limitations of this STD method, \( l_{st} < l_{str} \) and \( \Delta V_{str} < l_{str} \) (see the analysis detail in Appendix A), where \( l_{str} \) is the spacecraft separation scale, \( l_{st} \) is the structure’s scale, and \( \Delta \) is the time step which we used in calculating the \( \Delta \hat{B}/\Delta t \). Because the accuracy of the magnetic field, \( \delta B \) (where \( B \) is one component of the magnetic field), is limited, the field variation during the period of \( \Delta t \) should be larger than \( \delta B \), that is \( \delta B < \Delta \hat{B}/\Delta t \), the \( \Delta t \) should satisfy \( \Delta t > \delta B/\Delta \hat{B} \). Similarly, \( l_{st} \) should satisfy \( l_{st} > \delta B/\partial \hat{B}/\partial \hat{n} \), where \( \hat{n} \) is along a certain direction. Thus, the optimal \( \Delta t \) and \( l_{st} \) satisfy

\[
\frac{\delta B}{\Delta \hat{B}} < \Delta t < \frac{l_{str}}{V_{str}}, \quad (3)
\]

\[
\frac{\delta B}{\partial \hat{B}/\partial \hat{n}} < l_{st} < l_{str} \quad (4)
\]

Therefore, suitable \( \Delta t \) and \( l_{st} \) should be taken for different cases.

3. Applications of the Method

[8] We use Cluster magnetic field data [Balogh et al., 2001] and relative position data for the four spacecraft to illustrate the applications of the STD method. The estimate of \( \Delta \hat{B}/\Delta t \) is made by calculating the average difference of the magnetic field for the four spacecraft using central difference in time, where \( \Delta t \) is chosen to meet the requirement of Equation (3). The \( \nabla \hat{B} \) can be estimated by any of the methods cited in Section 2.

3.1. Velocity Vector Calculation for a 3-D Structure

[9] If the observed structure is 3-D, we can in principle obtain all three components of the velocity by directly solving the difference equations of Equation (1). At about 13:20 UT on 18 March, 2002, Cluster was moving outbound from the lobe to the cusp in the magnetosphere at \((-1.29, 2.49, 5.17)R_E \) for GSE, when all the four spacecraft observed an increase in the magnetic field. A time step \( \Delta t \) of 4 seconds is used here. Figure 1a shows that the structure is large compared to the spacecraft separation and all four spacecraft remain within it for the duration. Figure 1b shows us the calculated velocity at every moment. We find that during 13:22–13:28 UT, the \( V_z \) is near zero, while the \( V_x \)
and \( V_z \) vary from positive to negative. It can be inferred that this structure moved toward the sun, and then moved back. The motion possibly corresponds to the movement of the cusp as a whole (e.g., direct observation of this kind of cusp motion are reported by Zong et al. [2004]).

### 3.2. Velocity Calculation for a Quasi-2D Structure

Figure 2 shows an example of the velocity calculation for a 2-D structure. On 1 September, 2003, Cluster is near the magnetotail current sheet at \((-19, -1.2, -1.3) R_E\) in the GSE coordinate system. From the \( B_x \) plot in Figure 2a, one find that the four spacecraft traversed the current sheet at about 03:40/UT and \( B_x \) turned from negative to positive. The following four panels show the MDD analysis, from which we find that the structure is quasi-2-D (two \( \lambda \) values are much larger than the third one, see Figure 2b), and the invariant direction is along \( \langle \hat{n}_3 \rangle / |\langle \hat{n}_3 \rangle| = (-0.5225, 0.7562, 0.3939) \), while the intermediate and maximum direction is along \( \langle \hat{n}_2 \rangle / |\langle \hat{n}_2 \rangle| = (0.8244, 0.5660, 0.0054) \) and \( \langle \hat{n}_1 \rangle / |\langle \hat{n}_1 \rangle| = (0.2198, -0.3304, 0.9179) \), respectively (see Figures 2c–2e). The time step \( \Delta t \) we use here is also 4 seconds. For the reason discussed above in Section 2, the projection of the calculated velocity (by solving the equations of Equation (1)) to the \( \hat{n}_1 \) and \( \hat{n}_2 \) directions at every moment give stable velocity components \( (V_{n1}, V_{n2}) \) along these two directions. The third component is anomalous (not shown here), as we expected. For this event we applied both calculation ways discussed in Section 2 and get the same results. From the plot of \( V_{n1} \) in Figure 2f, we find that the current sheet flipped down and then up, which corresponds well to the variation of \( B_x \). The maximum velocity along \( \hat{n}_1 \) (close to \( z \)) is 15.4 km/s. From the plot of \( V_{n2} \), we find that the flipping velocity also has the component perpendicular to the \( \tilde{z} \) axis (see \( \hat{n}_2 \)), with the maximum along \( \hat{n}_2 \) is 11.6 km/s. We leave further detailed interpretation to future study, however.

In principle, all the existed multi-point methods cited in Section 1 (such as Timing or CVA, DA, CTA and so on) could not calculate more than one components of the velocity for 3-D and 2-D structures. They can only calculate the velocity along the normals for pre-assumed 1-D structures. So the STD method may supply this gap.

### 3.3. Velocity Calculation for Quasi-1-D Structures (Planar Boundaries): Comparison to Other Methods

In this part we compare our results with other techniques using three dayside boundary crossings (treated as 1-D structures). Figures 3a and 3b show our calculation for the magnetopause crossing on 2 March, 2002 [Haaland et al., 2004]. Figure 3c shows the velocity along \( \hat{n}_1 \) (the bar gives the range of the results given by Haaland et al. [2004]). (c–f) The STD analysis results for two 1-D cusp boundaries around 21:41 UT and 21:45 UT, Feb. 4, 2001: Figure 3e shows the maximum derivative direction \( \hat{n}_1 \) in GSE (dashed lines are the normal calculated by timing method); Figure 3d shows the velocity along \( \hat{n}_1 \) (the bar gives the range obtained by timing and DA method). Figures 3e and 3f have the same formats with Figure 3c and 3d.
et al., 2004]. The time step $\Delta t$ used in our calculation is 1 second (note that it is different from the time resolution of the data: 1/3 s). From the MDD analysis [see Shi et al., 2005], we have confirmed this structure to be 1-D. For this crossing, Haaland et al. [2004] have applied several methods including MVAJ, CVA, CTA, HT combined with MVAB, and MFR to calculate both the boundary normal and its velocity. They find velocity results, which vary from $-42.1$ to $-30.5$ km/s (the square in Figure 3b) along the normal (the dashed lines in Figure 3a). We find the average normal (or the variation direction) is close to the normal given by Haaland et al. [2004], seeing shaded area (where all the 4 spacecraft are in the same layer and the gradient estimating is valid, the same below) during 03:31:08–03:31:13 in Figure 3a. The velocity shown in Figure 3b is along $\vec{n}_1$ and ranges from $-36.2$ to $-21.6$ km/s, pointing into the magnetosphere, agreeing well with the range found by Haaland et al. [2004].

[13] Figures 3c–3f show us two other boundaries in the high-altitude cusp at about 21:41 UT and 21:45 UT on 04 February, 2001, [studied by Lavraud et al., 2002]. For the first boundary (see shading areas in Figures 3c and 3d), we see that the MDD normal slightly changes with respect to that calculated by the timing method (dashed lines). The velocity is close to that of the DA and timing method (solid bar in Figure 3d). For the second boundary (see shading areas in Figures 3e and 3f), we find that the normal is very close to the normal from the timing method, but the velocity is not stable. During the traversal we find the acceleration, which suggests the boundary undergoes an oscillation. Using DA method, we can also see an indication of this velocity variation (not shown here).

[14] All the comparisons above show that the STD method is feasible at the velocity calculation on the 1-D structures, and some details of the motion can be seen very easily.

4. Discussion

[15] We have shown the feasibility of the STD method using Cluster data for 3-D, 2-D, and 1-D events. For 1-D cases, our results have been compared to other boundary analysis methods. The advantage of the STD method is significant: we can obtain the velocity for the structures in any dimensionality, and can clearly see the time variation of the velocity clearly. This differs from most of the other existing techniques. It is very easy to apply and compute automatically. So it may be used to deal with a large number of events.

[16] As mentioned in section 2, care must be taken when using this method. So there are a number of caveats to be observed, which will be summarized here. A suitable $\Delta t$ should be taken considering the time scale of the structure (Equation (3)). All the spacecraft should remain within the structure with a suitable separation when we calculate the field gradient (Equation (4)). This imposed constraints on the relative time and spatial scales and can be tested by inspection of the time series. An additional judgment of the suitable calculation can be made by comparing the $\Delta B/\Delta t$ estimated for all the spacecraft, which will be detailed in the future. In the velocity calculation, we have to adopt the assumption of stationarity, although this should be satisfied in many cases selected. A method may therefore be needed in the future to separate the non-steady components of the structure or estimate their influence on the results.

Appendix A: The Truncation Errors in the Difference Equations of Equation (1)

[17] For a 1-D case, assuming the magnetic field is along $x$ and the motion is along $z$, we need to solve

$$\frac{dB_x}{dt} = -V_z \frac{\partial B_x}{\partial z}. \quad (A1)$$

Supposing two spacecraft are aligned along $z$, we get the difference equation of (A1) by using central (2-order) difference in time and 1-order difference in space, $(B_{x1}^{n+1} + B_{x1}^{n-1})/(2\Delta t) = -V_z(B_{x1}^n - B_{x1}^{n-1})/\Delta z$. Applying Taylor expansion in the vicinity of $B_{x1}^n$, we get

$$\frac{dB_{x1}}{dt} + 1/6(\Delta t)^2 \frac{\partial^3 B_{x1}}{\partial z^3} + O(\Delta t^4)$$

$$= -V_z [\partial B_{x1}/\partial z + 1/2\Delta z (\partial^2 B_{x1}/\partial z^2 + O(\Delta z^2))]. \quad (A2)$$

If $V_z$ is constant, from Equation (A1) we get $\frac{d^2 B_x}{dt^2} = V_z^2 \frac{\partial^2 B_x}{\partial z^2}$, and $\frac{d^2 B_x}{dt^2} = -\frac{\partial}{\partial z} B_x^2 \frac{\partial}{\partial z} z$. Substitute them and (A1) to Equation (A2) and omitting the high order terms, we get

$$-V_z \frac{\partial B_{x1}/\partial z}{\partial z} - 1/6(\Delta t)^2 V_z^2 \frac{\partial^3 B_{x1}}{\partial z^3}$$

$$= -V_z \frac{\partial B_{x1}/\partial z}{\partial z} + 1/2(\Delta t)^2 \frac{\partial^3 B_{x1}}{\partial z^3}. \quad (A3)$$

Comparing the error term and the original term in the left side, we get 1/6$(\Delta t^2/L_{str}^2)$, which should be much smaller than 1. So, the $\Delta t$ should satisfy $\Delta t < L_{str}$. Similarly, from the right side, we get 1/2$\Delta z/L_{str}$ < 1. Hence the limitation of $\Delta z$ is $\Delta z < L_{str}$. Generalizing these limitations directly to the 3-D case, we then get $L_{str} < L_{str}$ and $\Delta t < L_{str}$. $\Delta V_{str} < L_{str}$.


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