

PITCH-ANGLE DIFFUSION COEFFICIENTS OF CHARGED PARTICLES FROM COMPUTER SIMULATIONS

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ABSTRACT

Pitch-angle diffusion is a key process in the theory of charged particle scattering by turbulent magnetic plasmas. This process is usually assumed to be diffusive and can, therefore, be described by a pitch-angle diffusion or Fokker–Planck coefficient. This parameter controls the parallel spatial diffusion coefficient as well as the parallel mean free path of charged particles. In the present paper, we determine pitch-angle diffusion coefficients from numerical computer simulations. These results are then compared with results from analytical theories. Especially, we compare the simulations with quasilinear, second-order, and weakly nonlinear diffusion coefficients. Such a comparison allows the test of previous theories and will lead to an improved understanding of the mechanism of particle scattering.

Key words: diffusion – magnetic fields – turbulence

1. INTRODUCTION

A fundamental problem in astro-, plasma-, and geophysics is the propagation of charged particles (e.g., cosmic rays). A standard assumption of transport theory is that particles propagate diffusively along and across a mean magnetic field. In the recent years, however, several authors have discovered non-diffusive particle transport (see, e.g., Zimbardo et al. 2006, 2008; Shalchi & Kourakis 2007; Perri & Zimbardo 2007). The parallel mean free path λ_{\parallel} of the charged particles is related to the parallel spatial diffusion coefficient κ_{\parallel} via $\lambda_{\parallel} = 3\kappa_{\parallel}/v$ and can be expressed as an integral over the inverse pitch-angle Fokker–Planck coefficient $D_{\mu\mu}$ (see, e.g., Earl 1974),

$$\lambda_{\parallel} = \frac{3v}{8} \int_{-1}^{+1} d\mu \frac{(1 - \mu^2)^2}{D_{\mu\mu}(\mu)}, \quad (1)$$

with the pitch-angle cosine $\mu = v_{\parallel}/v$ and the particle velocity v .

The first approach for computing the parameter $D_{\mu\mu}$ was the application of perturbation theory also known as quasilinear theory (QLT; Jokipii 1966). As explained in Shalchi (2009), there are three major problems in transport theory, namely:

1. *The 90° problem.* QLT does not provide the correct result for pitch-angle diffusion for pitch angles close to 90° ($\mu = 0$). The 90° problem was discovered in the years after QLT has been proposed. Therefore, nonlinear theories were developed, most of them are extensions of QLT. Some examples are the nonlinear perturbation theory (Völk 1973), the partially averaged field theory (Jones et al. 1973, 1978), the nonlinear closure approximation (Owens 1974), the *Heuristic Ansatz* of Völk (1975), and the strong turbulence weak coupling theory (Goldstein 1976). More recently a second-order quasilinear theory (SOQLT) has been developed by Shalchi (2005a).
2. *The problem of perpendicular diffusion.* QLT cannot describe particle transport perpendicular to a mean magnetic field. Several theories have been derived for improving the description of cross-field diffusion. Some examples are the

nonlinear closure approximation (Owens 1974), the compound diffusion model (see, e.g., Urch 1977; Kóta & Jokipii 2000; Shalchi 2005b; Webb et al. 2006), and the nonlinear guiding center theory (Matthaeus et al. 2003). The latter theory was improved/extended by several authors (see, e.g., Shalchi 2006; Qin 2007; Shalchi & Dosch 2008, 2009).

3. *The geometry problem.* QLT disagrees with simulations for parallel diffusion if a non-slab turbulence geometry is assumed. This third problem of particle diffusion theory has been discovered by Minnie (2002) and was also solved by using nonlinear diffusion theories. The first theory which has shown agreement with simulated parallel mean free paths for non-slab turbulence was the weakly nonlinear theory (WNLT; Shalchi et al. 2004). Le Roux & Webb (2007) have confirmed the weakly nonlinear theory by using a BGK–Boltzmann approach. Later, Qin (2007) has used an extension of the nonlinear guiding center theory to reproduce these simulations too.

In the present paper we will focus on the first of these three problems, namely, on the 90° problem. We will use a test-particle code which has been used previously for calculating charged particle orbits and particle diffusion coefficients (see, e.g., Mace et al. 2000; Qin 2002; Qin et al. 2002a, 2002b, 2006) to compute the pitch-angle Fokker–Planck coefficient $D_{\mu\mu}$. In the following, we will compare the simulations with theoretical diffusion coefficients obtained by employing QLT as well as with the SOQLT of Shalchi (2005a). The latter theory is an extension of QLT and was derived for so-called slab turbulence. Later, the theory has successfully been reformulated for isotropic turbulence by Tautz et al. (2008). Some results are also compared with the weakly nonlinear theory of Shalchi et al. (2004) and the predictions made by the theory of Shalchi et al. (2008).

2. ANALYTICAL THEORIES FOR PITCH-ANGLE DIFFUSION

In the case of diffusive transport, the pitch-angle Fokker–Planck coefficient $D_{\mu\mu}$ can be computed by employing the so-called Taylor–Green–Kubo (TGK) formulation (e.g., Taylor 1922; Green 1951; Kubo 1957):

$$D_{\mu\mu} = \int_0^{\infty} dt \langle \dot{\mu}(t)\dot{\mu}(0) \rangle. \quad (2)$$

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The acceleration parameter $\dot{\mu}(t)$ can be obtained from the Newton–Lorentz equation whose parallel component reads

$$\dot{\mu} = \frac{1}{R_L} \left(v_x(t) \frac{\delta B_y[\vec{x}(t), t]}{B_0} - v_y(t) \frac{\delta B_x[\vec{x}(t), t]}{B_0} \right) \quad (3)$$

for purely magnetic fields. Here, we have used the unperturbed Larmor radius

$$R_L = \frac{v}{\Omega} = \frac{vmc\gamma}{qB_0} = \frac{pc}{qB_0}, \quad (4)$$

with the unperturbed gyrofrequency Ω , the particle mass m and charge q , the mean magnetic field B_0 , the speed of light c , and the particle momentum p . In the following sections, we will briefly discuss two approaches to evaluate Equations (2) and (3).

2.1. Standard Quasilinear Theory

The simplest method to compute the parameter $D_{\mu\mu}$ is the application of QLT (Jokipii 1966). In this case, the velocities $v_x(t)$ and $v_y(t)$ as well as the particle trajectories $\vec{x}(t)$ in Equation (3) are replaced by the unperturbed particle orbit. A further assumption which is often used is that the stochastic magnetic fields δB_i are replaced by the so-called magnetostatic slab model for which we assume $\delta B_i(\vec{x}, t) = \delta B_i(z)$, where z is the (Cartesian) coordinate along the mean field B_0 . For the slab model, the magnetic correlation tensor is given by

$$P_{ij}^{\text{slab}}(\vec{k}) \equiv \langle \delta B_i(\vec{k}) \delta B_j^*(\vec{k}) \rangle = g^{\text{slab}}(k_{\parallel}) \frac{\delta(k_{\perp})}{k_{\perp}} \delta_{ij}, \quad (5)$$

with the (symmetric) turbulence spectrum $g^{\text{slab}}(k_{\parallel})$. The combination of QLT and the magnetostatic slab model is also known as standard QLT. In this case, we find (see, e.g., Shalchi 2009 for a detailed derivation of this formula)

$$D_{\mu\mu} = \frac{2\pi v^2(1 - \mu^2)}{B_0^2 R_L^2} \int_0^{\infty} dk_{\parallel} g^{\text{slab}}(k_{\parallel}) \times [R_+(k_{\parallel}) + R_-(k_{\parallel})], \quad (6)$$

with the quasilinear resonance function

$$R_{\pm}^{\text{QLT}}(k_{\parallel}) = \pi \delta(v\mu k_{\parallel} \pm \Omega). \quad (7)$$

All parameters used in the present paper are explained in Table 1. The particle experiences only interaction if the gyroresonance condition $v\mu k_{\parallel} = \Omega$ (corresponding to $\mu R_L k_{\parallel} = 1$) is fulfilled. We find for the quasilinear pitch-angle Fokker–Planck coefficient

$$D_{\mu\mu}^{\text{QLT}} = \frac{2\pi^2 v(1 - \mu^2)}{|\mu| B_0^2 R_L^2} g^{\text{slab}}[k_{\parallel} = (|\mu| R_L)^{-1}]. \quad (8)$$

In combination with Equation (1) this formula can be used to compute the quasilinear parallel mean free path requiring knowledge of the slab spectrum $g^{\text{slab}}(k_{\parallel})$.

2.2. Second-order Quasilinear Theory

In the SOQLT of Shalchi (2005a) we no longer assume unperturbed orbits. Instead, QLT is employed in order to compute improved orbits. The improved orbits are then combined with

Table 1
Parameters and their Standard Values Used in the Present Paper

Parameter	Symbol	Standard Value
Inertial range spectral index	s	5/3
Energy range spectral index	q	0
Slab bendover scale	l_{slab}	0.03 AU
Two-dimensional bendover scale	l_{2D}	$0.1 l_{\text{slab}} = 0.003 \text{ AU}$
Particle velocity	v	...
Pitch-angle cosine	μ	$-1 \leq \mu \leq 1$
Larmor radius	R_L	...
Gyrofrequency	Ω	...
Parallel wavenumber	k_{\parallel}	...
Perpendicular wavenumber	k_{\perp}	...
Mean magnetic field	B_0	4 nT
Turbulence strength	$\delta B/B_0$	Variable
Energy in the slab modes	$\delta B_{\text{slab}}^2/B_0^2$	Variable
Energy in the two-dimensional modes	$\delta B_{2D}^2/B_0^2$	Variable
Dimensionless rigidity	$R = R_L/l_{\text{slab}}$	Variable
Dimensionless time	$\tau = vt/l_{\text{slab}}$	Variable

Equations (2) and (3). Mathematically, the second-order approach leads to a broadened resonance function (for a detailed derivation see Shalchi 2005a, 2009):

$$R_{\pm}^{\text{SOQLT}}(k_{\parallel}) = \int_0^{\infty} dt \cos[(k_{\parallel} v \mu + n \Omega) t] \times e^{-v^2 k_{\perp}^2 \int_0^t d\tau [\tau^2 D_{\mu\mu}^{\text{QLT}}(\tau)]}, \quad (9)$$

with the time-dependent quasilinear Fokker–Planck coefficient $D_{\mu\mu}^{\text{QLT}}(t)$. By combining the second-order approach with two mathematical approximations, namely a *late-time approximation (LTA)* and a *90° approximation*, Shalchi (2005a) has derived the following formula:

$$R_{\pm}^{\text{SOQLT}}(k_{\parallel}) = \frac{\sqrt{\pi} B_0}{v k_{\parallel} \delta B} \exp \left[-\frac{(v\mu k_{\parallel} \pm \Omega)^2 B_0^2}{(v k_{\parallel} \delta B)^2} \right]. \quad (10)$$

Here, the resonance function has the form of a Gaussian function. The quasilinear resonance function (see Equation (7)) can be recovered by considering the limit $\delta B^2/B_0^2 \rightarrow 0$ in Equation (10). The second-order pitch-angle Fokker–Planck coefficient can be calculated by combining Equations (6) and (10).

2.3. Analytical Second-order Results for a Specific Turbulence Spectrum

For the turbulence spectrum we employ the form proposed by Shalchi & Weinhorst (2009),

$$g^{\text{slab}}(k_{\parallel}) = \frac{D(s, q)}{2\pi} \delta B_{\text{slab}}^2 l_{\text{slab}} \frac{|k_{\parallel} l_{\text{slab}}|^q}{[1 + (k_{\parallel} l_{\text{slab}})^2]^{(s+q)/2}}, \quad (11)$$

with the normalization function

$$D(s, q) = \frac{\Gamma(\frac{s+q}{2})}{2\Gamma(\frac{s-1}{2})\Gamma(\frac{q+1}{2})}. \quad (12)$$

In Equation (11), we have used the slab bendover scale l_{slab} . This parameter denotes the frequency break between the large scales (energy range) and the intermediate scales (inertial range) of the turbulence. Furthermore, we have used the energy range spectral index q and the inertial range spectral index s . The spectrum is correctly normalized for $q > -1$ and $s > 1$.

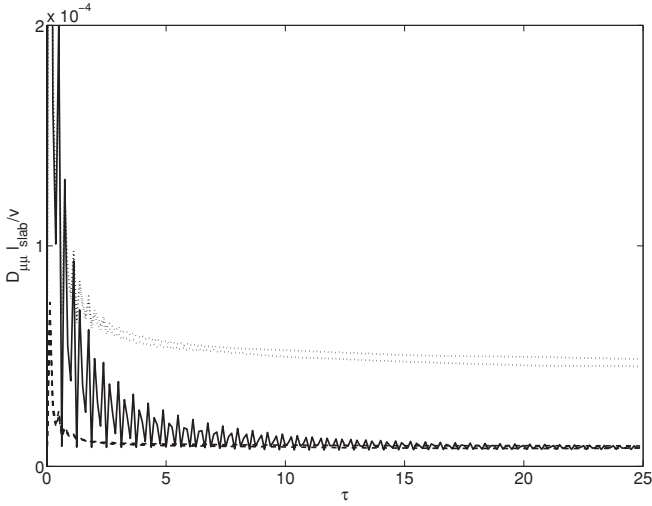


Figure 1. Pitch-angle Fokker–Planck coefficients vs. the dimensionless time $\tau = vt/l_{\text{slab}}$ for pure slab turbulence. Shown are the simulations for $R = 0.05$ and $\delta B/B_0 = 0.01$ for different values of μ , namely, $\mu = \pm 1$ (dashed lines), $\mu = \pm 0.5$ (dotted lines), and $\mu = 0$ (solid line).

Shalchi et al. (2009) have calculated the second-order pitch-angle Fokker–Planck coefficient analytically by using the spectrum defined in Equation (11). They found

$$D_{\mu\mu} = \frac{\pi D(s, q)}{2s} (1 - \mu^2) \frac{v}{l_{\text{slab}}} R^{s-2} \frac{\delta B}{B_0} \times \sum_{n=\pm 1} \text{sign} \left(\frac{\delta B}{B_0} + n |\mu| \right) \cdot \left| |\mu| + n \frac{\delta B}{B_0} \right|^s. \quad (13)$$

This formula can be applied for $R_L ||\mu| \pm \delta B/B_0| \ll l_{\text{slab}}$ corresponding to the case of not too large particle Larmor radii R_L .

3. PITCH-ANGLE DIFFUSION COEFFICIENTS FROM COMPUTER SIMULATIONS

Here, we present a comparison between the theoretical results discussed previously with numerical simulations. The simulation code used in this paper to calculate charged particle transport parameters has been used before (see, Mace et al. 2000; Qin 2002; Qin et al. 2002a, 2002b, 2006 for details). Test-particle simulations have also been performed by Kaiser (1975), Kaiser et al (1978), Michalek & Ostrowski (1996), Giacalone & Jokipii (1999), Zimbardo et al. (2006), and Tautz (2009a, 2009b). In our numerical simulations, the Newton–Lorentz equation is solved by a fourth-order Runge–Kutta method with adaptive step size control (see, e.g., Press et al. 1992).

In our simulations, the model magnetic field \vec{B} consists of a uniform mean magnetic field \vec{B}_0 in the z -direction and a composite model of transverse magnetic fluctuations consisting of a two-dimensional (2D) part $\delta B_{2D}(x, y)$ and slab modes $\delta B_{\text{slab}}(z)$. The turbulence level is controlled by the ratio $\delta B/B_0 = \sqrt{\delta B_{\text{slab}}^2 + \delta B_{2D}^2}/B_0$. The turbulence geometry is set by the ratio $E_{\text{slab}}/(E_{\text{slab}} + E_{2D})$ with $E_i \equiv \delta B_i^2$. For both components we have employed a flat spectrum when the wavenumber k is much smaller than the inverse bendover scale l_i^{-1} . For higher wavenumbers, we assumed a Kolmogorov (1941) spectrum with $k^{-5/3}$.

The slab spectrum is created with a periodic box of size $10,000l_{\text{slab}}$ and $N_z = 4194, 304$ points and the two-dimensional

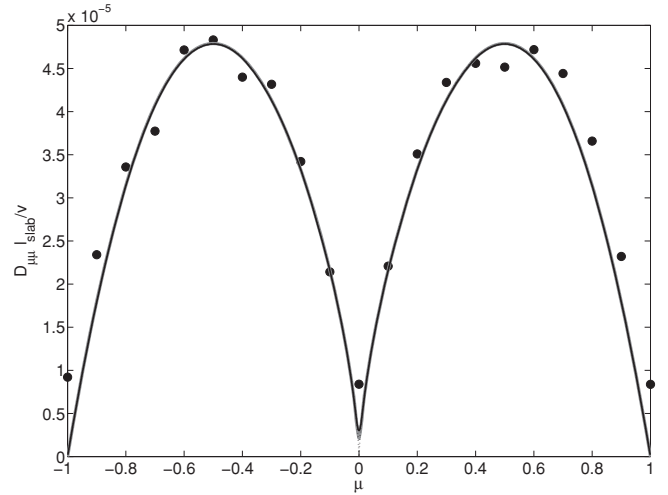


Figure 2. Pitch-angle Fokker–Planck coefficients for pure slab turbulence. Shown are the results of QLT (dotted line), SOQLT within the 90° LTA (solid line), and simulations (dots) for $D_{\mu\mu}$. Furthermore, we have shown the analytical result derived by using Equation (13) (gray line). In this plot, the rigidity is $R = 0.05$ and the strength of turbulence is $\delta B/B_0 = 0.01$. Note that some results are in coincidence.

spectrum is created with a periodic box of size $100l_{\text{slab}} \times 100l_{\text{slab}}$ and $N_x \times N_y = 4096 \times 4096$ points. The perpendicular correlation scale for two-dimensional turbulence, l_{2D} , is set to be 1/10 of the parallel correlation scale.

To study the pitch-angle diffusion coefficients, we calculate 10,000 test particle trajectories for each model turbulence configuration. From the trajectories, we calculate running pitch-angle diffusion coefficients $D_{\mu\mu}(\mu, t)$. If we get constant values of running pitch-angle diffusion coefficients, we consider to get true diffusion, i.e.,

$$D_{\mu\mu}(\mu) = \lim_{t \rightarrow \infty} D_{\mu\mu}(\mu, t). \quad (14)$$

3.1. Simulations for Pure Slab Turbulence

Here, we use the slab model ($\delta B \equiv \delta B_{\text{slab}}$) defined by Equation (5) and we set $q = 0$ and $s = 5/3$ in the spectrum defined in Equation (11). We performed the simulations for $\delta B/B_0 = 0.01$. We computed the time-dependent (running) pitch-angle diffusion coefficients for different values of the pitch-angle cosine μ . For the magnetic rigidity, we have used $R \equiv R_L/l_{\text{slab}} = 0.05$. The results are shown in Figure 1. We find that pitch-angle scattering is at least nearly diffusive.

In Figures 2–4, we have shown $D_{\mu\mu}$ versus the pitch-angle cosine μ . As demonstrated, QLT agrees with the simulations for nearly all values of μ . If we are close to 90° (corresponding to $\mu = 0$), however, QLT predicts vanishing pitch-angle scattering. This result disagrees with the simulations. If we replace QLT by the second-order theory, we find indeed strong scattering at 90° as predicted by the simulations. For increasing turbulence strength, the pitch-angle diffusion coefficients at 90° increases more rapidly than at other angles. This behavior is also in agreement with the second-order result.

3.2. Simulations for Pure Two-dimensional Turbulence

At least for interplanetary studies, a common assumption is that the magnetic field fluctuations admit a strong component of nearly two-dimensional character with $\delta \vec{B}(\vec{x}) = \delta \vec{B}(x, y)$ (see, e.g., Matthaeus et al. 1990; Zank & Matthaeus 1993). For the

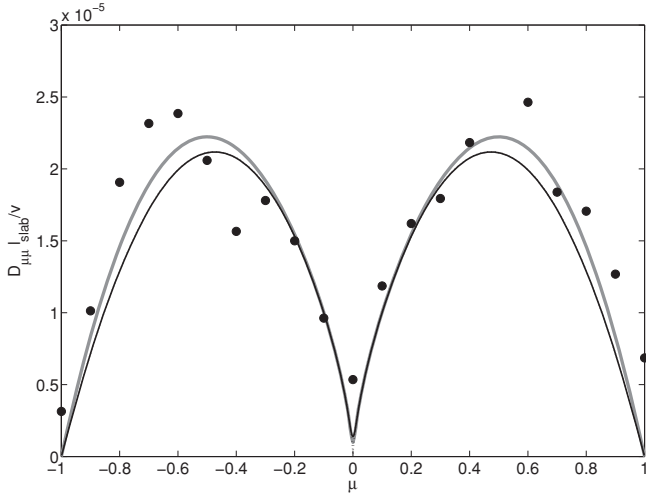


Figure 3. Pitch-angle Fokker–Planck coefficients for pure slab turbulence. Shown are the results of QLT (dotted line), SOQLT within the 90° LTA (solid line), and simulations (dots) for $D_{\mu\mu}$. Furthermore, we have shown the analytical result derived by using Equation (13) (gray line). In this plot, the rigidity is $R = 0.5$ and the strength of turbulence is $\delta B/B_0 = 0.01$. Note that some results are in coincidence.

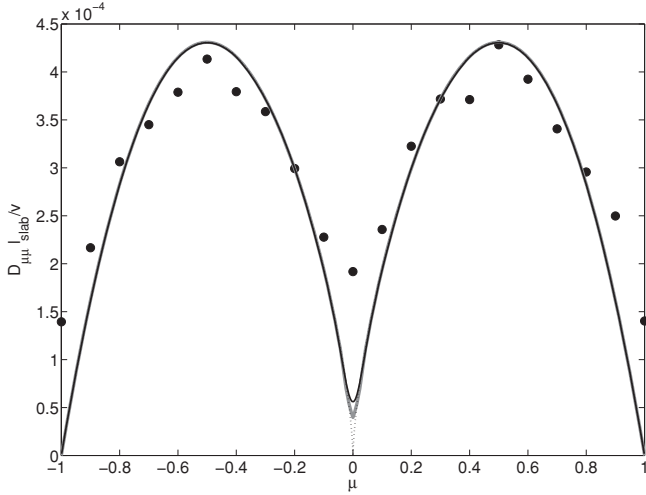


Figure 4. Pitch-angle Fokker–Planck coefficients for pure slab turbulence. Shown are the results of QLT (dotted line), SOQLT within the 90° LTA (solid line), and simulations (dots) for $D_{\mu\mu}$. Furthermore, we have shown the analytical result derived by using Equation (13) (gray line). In this plot, the rigidity is $R = 0.05$ and the strength of turbulence is $\delta B/B_0 = 0.03$. Note that some results are in coincidence.

two-dimensional model the magnetic correlation tensor is given by

$$P_{lm}^{2D}(\vec{k}) = g^{2D}(k_{\perp}) \frac{\delta(k_{\parallel})}{k_{\perp}} \left[\delta_{lm} - \frac{k_{\parallel} k_m}{k^2} \right], \quad l, m = x, y. \quad (15)$$

Here, we have used the turbulence spectrum of the two-dimensional modes $g^{2D}(k_{\perp})$. For the shape of the spectrum we used a form similar to Equation (11), namely,

$$g^{2D}(k_{\perp}) = \frac{2D(s, q)}{\pi} \delta B_{2D}^2 l_{2D} \times \frac{(k_{\perp} l_{2D})^q}{[1 + (k_{\perp} l_{2D})^2]^{(s+q)/2}}. \quad (16)$$

As shown by Shalchi et al. (2008), pitch-angle diffusion in pure two-dimensional turbulence should be subdiffusive with

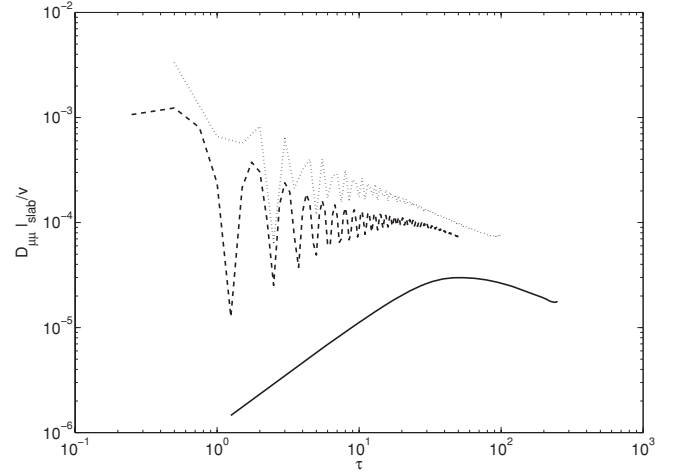


Figure 5. Pitch-angle Fokker–Planck coefficient $D_{\mu\mu}$ vs. the dimensionless time $\tau = vt/l_{slab}$ for pure two-dimensional turbulence. Shown are the simulations for $\mu = 0$ and $R = 0.05$. We have computed the time-dependent parameter $D_{\mu\mu}$ for $\delta B/B_0 = 0.01$ (solid line), $\delta B/B_0 = 0.03$ (dashed line), and $\delta B/B_0 = 0.05$ (dotted line).

$\langle(\Delta\mu)^2\rangle \sim \sqrt{t}$ in contradiction with the assumption of diffusive transport with $\langle(\Delta\mu)^2\rangle \sim t$. It is also the purpose of the present paper to check this prediction.

For the simulations we used $\delta B_{2D} = \delta B$, $s = 5/3$, $q = 0$, $l_{slab} = 0.030$ AU, and $l_{2D} = 0.1l_{slab} = 0.003$ AU. The results are visualized for $R = 0.05$ and $\mu = 0$ in Figure 5 as log–log plot for different turbulence strengths. Clearly, we see a subdiffusive behavior of transport. We have also performed the simulations for other values of μ . In this case, we found a weaker subdiffusivity. According to Shalchi et al. (2008), the strength of the non-diffusivity depends on the assumption about the perpendicular Fokker–Planck coefficient $D_{\perp}(\mu, t)$. Subdiffusive transport with $\langle(\Delta\mu)^2\rangle \sim \sqrt{t}$ can only be obtained for diffusive perpendicular scattering ($D_{\perp}(\mu, t) = D_{\perp}(\mu)$). For superdiffusive cross-field transport, the non-diffusivity of pitch-angle scattering becomes weaker.

3.3. Simulations for Slab/Two-dimensional Turbulence

In the solar wind it is common to assume a superposition of slab modes and two-dimensional modes (see, e.g., Matthaeus et al. 1990; Bieber et al. 1996). As a consequence, the turbulent field can be written as $\delta\vec{B}(\vec{x}) = \delta\vec{B}(z) + \delta\vec{B}(x, y)$. In the literature, this model is also known as slab/two-dimensional composite or two-component model. For this model, the magnetic correlation tensor in the wavevector space has the form $P_{lm} = P_{lm}^{slab} + P_{lm}^{2D}$. For the two correlation tensors we use Equations (5) and (15) and for the two wavenumbers Equations (11) and (16).

The simulations are done for $R = 0.05$, $\delta B_{slab}^2 = 0.2\delta B^2$, $\delta B_{2D}^2 = 0.8\delta B^2$, $s = 5/3$, $l_{slab} = 0.03$ AU, and $l_{2D} = 0.1l_{slab} = 0.003$ AU. The results are compared with quasilinear results but not with SOQLT, since the second-order theory has only been formulated for pure slab and isotropic turbulence. For completeness, we have also shown results of the weakly nonlinear theory. However, it was already noted in Shalchi (2005a) that WNLT is incorrect for 90° scattering.

In Figures 6 and 7, we have visualized the simulations and QLT results for $\delta B/B_0 = 0.01$ and $\delta B/B_0 = 0.03$. In Figure 8, we compare the simulations for two-component turbulence with the pure slab results for $\delta B/B_0 = 0.01$. As shown 90° scattering is very efficient in two-component turbulence and

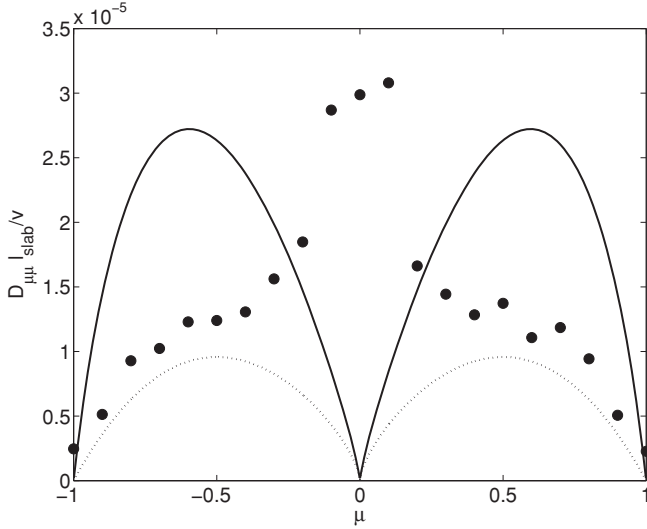


Figure 6. Pitch-angle Fokker–Planck coefficients for two-component turbulence with $\delta B_{\text{slab}}^2 = 0.2\delta B^2$. Shown are the results of QLT (dotted line), WNLT (solid line), and simulations (dots). In this plot, the rigidity is $R = 0.05$ and the strength of turbulence is $\delta B/B_0 = 0.01$.

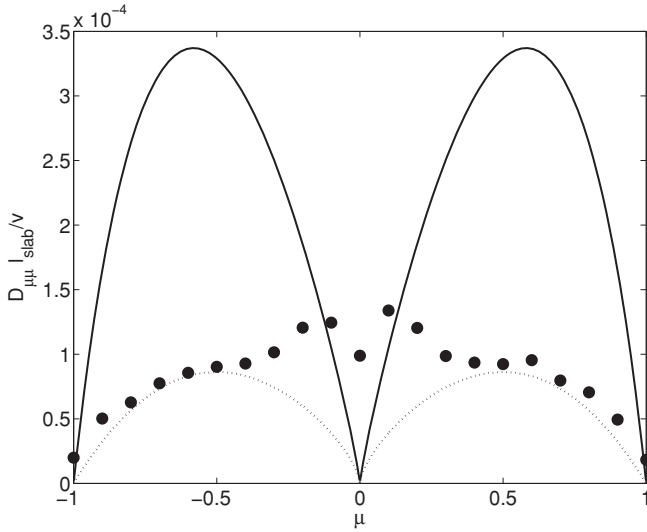


Figure 7. Pitch-angle Fokker–Planck coefficients for two-component turbulence with $\delta B_{\text{slab}}^2 = 0.2\delta B^2$. Shown are the results of QLT (dotted line), WNLT (solid line), and simulations (dots). In this plot, the rigidity is $R = 0.05$ and the strength of turbulence is $\delta B/B_0 = 0.03$.

more important than in the pure slab model for the considered parameter regime. As predicted, WNLT is incorrect for pitch angles close to 90° . Since we have discovered that 90° scattering is very efficient in two-component turbulence, it seems to be an interesting and important task to reformulate the SOQLT for the slab/two-dimensional model.

3.4. The Problem of Strong Turbulence

In the present paper, we investigate pitch-angle diffusion for weak turbulence ($\delta B \ll B_0$). The considered values are not in agreement with solar wind observations which indicate that the turbulence field can approach or exceed the strength of the mean magnetic field. The reason for considering weak turbulence is that the simulations cannot be performed for stronger turbulence. For strong turbulence we were not able to compute the parameter $D_{\mu\mu}(\mu)$ in the stable regime ($t \rightarrow \infty$) since the pitch-angle changes permanently. In Figure 9, we have shown $\mu(t)$ for a typical particle which experiences pitch-

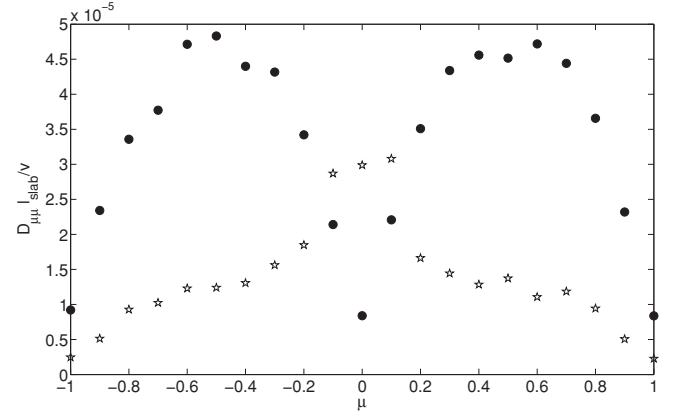


Figure 8. Pitch-angle Fokker–Planck coefficients for two-component turbulence with $\delta B/B_0 = 0.01$. Compared are the simulations for two-component turbulence ($\delta B_{\text{slab}}^2 = 0.2\delta B^2$, stars) and pure slab turbulence ($\delta B_{\text{slab}}^2 = 1.0\delta B^2$, dots).

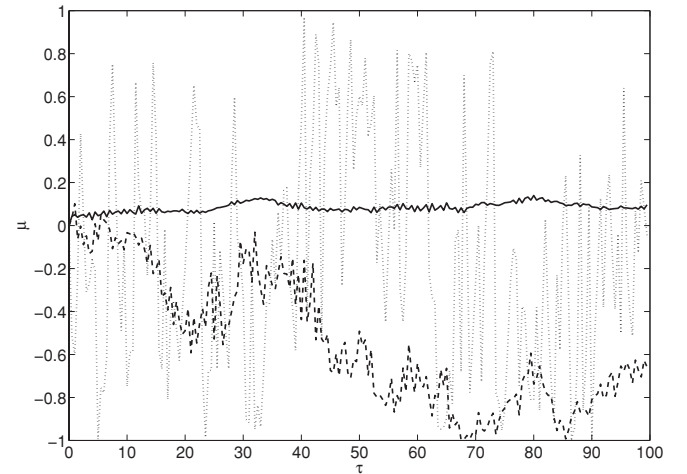


Figure 9. Particle's pitch-angle cosine μ for pure slab turbulence and $R = 0.05$. Shown are the results for different turbulence strength, namely, for $\delta B/B_0 = 0.01$ (solid line), $\delta B/B_0 = 0.1$ (dashed line), and $\delta B/B_0 = 1.0$ (dotted line).

angle scattering. For weak turbulence ($\delta B/B_0 = 0.01$), we have $\mu \approx \text{const}$ and, therefore, we are able to compute the pitch-angle Fokker–Planck coefficient as a function of μ . For strong turbulence pitch-angle scattering is so efficient that we cannot compute $D_{\mu\mu}$ as a function of μ in the stable regime since μ exhibits strong fluctuations.

4. SUMMARY AND CONCLUSION

In the present paper, we used computer simulations to compute pitch-angle diffusion (or Fokker–Planck) coefficients $D_{\mu\mu}$. This is important since this coefficient controls the parallel spatial diffusion coefficient as well as the parallel mean free path—see Equation (1). In the past few years, several nonlinear diffusion theories have been developed to achieve a more accurate description of particle diffusion. One example is the SOQLT (Shalchi 2005a) which is an extension of QLT (Jokipii 1966). A theory for pitch-angle diffusion in pure two-dimensional turbulence has been developed by Shalchi et al. (2008). This approach predicts a subdiffusive behavior of pitch-angle scattering.

It is the purpose of the present paper to check some of the predictions made by these theories. Therefore, we have employed a test-particle code used previously to compute parallel and perpendicular mean free paths (see Mace et al. 2000; Qin 2002; Qin et al. 2002a, 2002b, 2006). This code has been

modified to compute pitch-angle Fokker–Planck coefficients $D_{\mu\mu}$. By comparing the simulations with different theoretical results, we have explored the validity of analytical theories. We have considered pure slab, pure two-dimensional, and two-component turbulence. The results are shown in Figures 1–9. The most important conclusions of the present paper are as follows.

1. A pitch-angle diffusion (or Fokker–Planck) coefficient has to be computed in the stable regime corresponding to the formal limit $t \rightarrow \infty$. For earlier times pitch-angle scattering is not diffusive (see Figure 1).
2. For stronger turbulence the pitch-angle changes rapidly (see Figure 9) and, therefore, the parameter μ is far away from being constant. In this case, we cannot compute the pitch-angle diffusion coefficient as a function of μ . Therefore, the parameter $D_{\mu\mu}$ can only be obtained from simulations in the limit of weak turbulence.
3. In Figures 2–4, we have performed test-particle simulations for pure slab turbulence and we have compared our results with quasilinear and second-order results. As shown we find strong scattering of charged particles at 90° . The second-order results are close to the simulations. QLT, however, predicts $D_{\mu\mu}(\mu = 0) = 0$ in disagreement with the simulations. This discrepancy is known as the 90° problem of scattering theory and leads to an infinitely large parallel mean free path for certain turbulence models (steep turbulence spectra, isotropic turbulence).
4. For pure two-dimensional turbulence, the theory of Shalchi et al. (2008) predicts subdiffusive pitch-angle scattering. The predication was confirmed in the present paper by using computer simulations (see Figure 5).
5. We have also performed simulations for two-component turbulence which should be a realistic model for solar wind turbulence (see, e.g., Matthaeus et al. 1990; Zank & Matthaeus 1993; Bieber et al. 1996). As demonstrated, 90° scattering becomes even more important if we merge from pure slab to two-component turbulence (see Figures 6–8). As already noted in Shalchi (2005a), WNLT cannot describe 90° scattering correctly. In the present paper, we proof this statement by comparing with simulations (see Figures 6 and 7).

As discovered in the present paper, 90° scattering is a very important effect. It is well known that 90° scattering is important for pure slab and strong turbulence (see, e.g., Shalchi 2005a, 2009). In the present paper, we have discovered that scattering through $\mu = 0$ is also very important if the slab model is replaced by the two-component model which should be a realistic model for solar wind turbulence. An analytical theory for 90° scattering in two-component turbulence is not available. The second-order theory was derived for pure slab turbulence and was later reformulated for isotropic turbulence. The weakly nonlinear theory cannot describe 90° scattering. Therefore, it seems to be important to develop an analytical theory for slab/two-dimensional turbulence which is valid for all pitch angles especially at 90° . This task will be subject of future work.

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