Transport in random magnetic fields: diffusion, subdiffusion and nonlinear second diffusion

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Abstract. We present numerical results that show, first, conditions under which parallel scattering reduces the effectiveness of perpendicular scattering, leading to the phenomenon of subdiffusion ("compound diffusion"), and second, that when sufficiently strong three dimensional effects are present, true diffusion is restored, with a suppressed perpendicular diffusion coefficient that depends upon the parallel mean free path.

INTRODUCTION

Standard quasilinear scattering theory [5] provides a framework for understanding transport of charged particles in directions both parallel to, and perpendicular to, a mean magnetic field. In this approximation the two effects are necessarily distinct from one another. In reality there should be connections between parallel and perpendicular transport, and there have been suggestions concerning its nature [6, 16, 14]; however as yet these effects are incompletely understood, and the realm of different theoretical formulations has not been well established. Here we describe two effects, each of which represents departures from quasilinear theory, and which involves interplay between parallel and perpendicular transport. First, recent numerical results [13] have demonstrated that scattering of charged particles in the direction parallel to the mean magnetic field can suppress diffusive transport perpendicular to the magnetic field. In such cases transport across the mean field is subdiffusive. Second, we show, again using numerical simulations, that a regime of second diffusion can be recovered, provided that the transverse structure of the turbulence is of sufficient complexity.

Diffusion of charged particles in directions perpendicular to the large scale average magnetic field remains incompletely understood [3, 11]. Heliospheric observations relating to perpendicular transport are puzzling, as on the one hand persistence of sharp boundaries [12] suggests a diminished role of transverse diffusion, while other evidence indicates enhanced transport of charged particles to widely separated latitudes [9],requiring robust transport across the mean interplanetary spiral magnetic field. A physically appealing picture is based upon the tendency of charged particles (or, more precisely, particle gyrocenters) to follow magnetic field lines. Field lines follow a random walk, and therefore so do the particles. However, fundamental questions can be raised regarding the applicability of this Field Line Random Walk (FLRW) limit to particle transport in certain geometries, and especially when there are one or more ignorable coordinates [7]. It is also troubling that numerical computations of low energy particle transport have so far failed to confirm FLRW behavior, or for that matter, any articulated theory of perpendicular diffusive transport [3, 11].

One possible complication is that perpendicular transport might not be a diffusion process. There are at least two ways in which this can occur. First, charged particles can be trapped and therefore have bounded displacements. A variation on this is the idea that in some circumstances the field lines themselves are non-diffusive [e.g., 4, 17]. Particles trying to follow such trapped or bounded field lines would then be restricted to non-diffusive transport. A second major possibility is that parallel scattering causes charged particles to scatter back along the same or a similar field line - by "retracing their steps" the particles experience a reduction in the rate of increase of perpendicular displacements [16]. In this "compound" transport scenario, particle transport is relegated to a subdiffusive rate even if the field lines are globally diffusive. Below we review an example of the type of numerical result [13] that verifies the compound subdif-

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fusive phenomenon, which occurs for model magnetic fluctuations that are only weakly dependent on the transverse coordinates. In this regard, we can examine the discussions of compound subdiffusion [16, 8, 13] which in essence hold that the accumulation of mean square perpendicular displacement is suppressed when parallel scattering reverses particle guiding center trajectories relative to a field line. On the other hand, the particles actually sample a bundle of field lines, and these may not be identical. As simulations verify, subdiffusion ensues when the field lines sampled within this field line bundle are almost identical. However under very similar circumstances the particles might well become randomized if the field lines sampled by the particles' gyro orbits have substantial dissimilarities. Thus it is a reasonable hypothesis that perpendicular diffusion is recovered if the magnetic field lines have sufficient transverse structure. We now proceed to demonstrate both subdiffusion and the latter effect – the recovery of diffusion due to nonlinear effects.

NUMERICAL SIMULATIONS

We compute test particle trajectories in magnetic turbulence with an adaptive step fourth order Runge Kutta method with a normalized per-step-accuracy of one part in 10^9. The particles with mass m and velocity $\vec{v}$ obey

$$m\frac{d\mathbf{v}(t)}{dt} = \frac{q\mathbf{v}(t)}{c} \times \mathbf{B}(\mathbf{X}), \tag{1}$$

where the laboratory frame electric field is neglected. The model magnetic field $\mathbf{B} = \mathbf{B}_0 + \mathbf{b}$ consists of a uniform mean magnetic field $\mathbf{B}_0$ (in the cartesian z-direction) to which is added a broad band spectrum of magnetic fluctuations. We choose a composite transverse magnetic fluctuation model $\mathbf{b} = (b_x(x,y,z), b_y(x,y,z), 0)$ consisting of a two dimensional (2D) part $\mathbf{b}^{2D}(x,y)$ and a one dimensional "slab" part $\mathbf{b}^{slab}(z)$. We control the amount of transverse structure by varying the ratio $E^{slab} : E^{2D}$. For more details see Mace et al. [11] and Qin et al. [13].

We calculate simultaneously the perpendicular and parallel diffusion coefficients employing the computed trajectories of charged particles. The calculations extend for time scales up to one thousand vt/λ_c (particle speed × time/correlation length). Both slab and 2D spectra become flat at wavenumbers k much less than the correlation scale and goes over to a $k^{-5/3}$ form at high k. We compute the running diffusion coefficients $\tilde{\kappa}_{xx} = \mathrm{d}\langle(\Delta \mathrm{x})^2\rangle/2\mathrm{dt}$ and $\tilde{\kappa}_{zz} = \mathrm{d}\langle(\Delta \mathrm{z})^2\rangle/2\mathrm{dt}$, where the time derivative is computed using a first order finite difference. When the mean square displacements are diffusive ($\propto t$), the running diffusion coefficient is identical to the usual one.

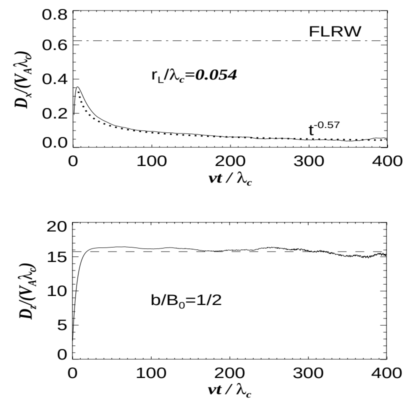

FIGURE 1. Perpendicular diffusion (upper panel) and parallel diffusion (lower panel) in nearly pure slab turbulent magnetic field ($E^{slab} : E^{2D} = 9999 : 1$) with low particle energy, $r_L/\lambda_c = 0.054$ and strong turbulence level, $b/B_0 = 0.5$. (Bottom panel) Solid line is running parallel diffusion coefficient vs. time. Dashed line is quasilinear theory result. (Top panel) Solid line is running perpendicular diffusion coefficient. Dotted line corresponds to subdiffusion. FLRW theoretical result is also shown. The running perpendicular diffusion coefficient $\sim t^{-0.57}$ in range vt/λ_c:[20,400].

NUMERICAL RESULTS

First we study test charged particle transport in nearly pure slab turbulent magnetic field, $E^{slab} : E^{2D} = 9999 : 1$ with large fluctuation amplitudes $b/B_0 = 1/2$ and small perpendicular correlation scale $\lambda_x = 0.1\lambda$, periodic box dimension, $L_z = 10000\lambda$, and the grids $N_z = 2^{22} = 4194304$.

The results are shown in Figure 1. The solid lines show perpendicular running diffusion (top panel) and parallel running diffusion (bottom panel). We can see at very short time scale running parallel diffusion performs free streaming while running perpendicular diffusion reaches its maximum value and then begins decreasing. This maximum value can be called first standard perpendicular diffusion. Then the running parallel diffusion settles into a constant value at a time scale when most particles have traveled a parallel mean free path. After parallel diffusion sets in the running perpendicular diffusion coefficient behaves very nearly as $\kappa_{xx} \propto 1/t^{1/2}$ which agrees with subdiffusion [16, 8].

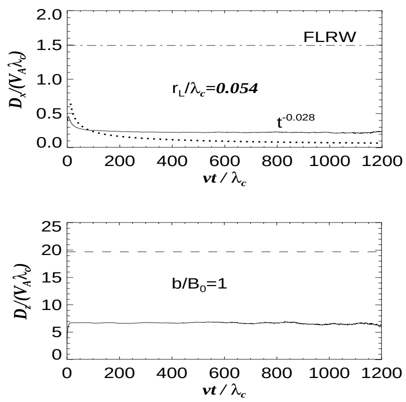

FIGURE 2. Diffusions in composite turbulent magnetic field ($E^{slab} : E^{2D} = 20 : 80$) with low particle energy, $r_L/\lambda_c = 0.054$ and strong turbulence level, $b/B_0 = 1$. The running perpendicular diffusion coefficient $\sim t^{-0.028}$ in range vt/λ_c:[200,1200] so the second regime of perpendicular diffusion is recovered.

The above example is typical of cases in which the turbulence is mostly of the slab type, so that the fluctuations vary weakly in the transverse directions. Nevertheless the turbulence field has no strictly ignorable coordinate [7]. The weak 2D fluctuations have a relatively small perpendicular correlation scale, and this does not favor the appearance of subdiffusion (see below). However, the result shows that on balance the particles in this example have been able to retrace their steps, thus suppressing perpendicular transport.

Now we turn to test particle diffusion in composite turbulent magnetic field with much stronger transverse structure. The spectrum of the slab and 2D model components now is chosen to have much more energy in the 2D component. In particular, we chose $E_{slab} : E_{2D} = 20 : 80$, large fluctuation amplitudes $b^2/B_0^2 = 1$, and small perpendicular correlation scale $\lambda_x = 0.1\lambda$. The periodic box has dimension $L_z = 10000\lambda$ and $L_x = L_y = 100\lambda$. The grids are $N_z = 2^{22} = 4194304$ and $N_x = N_y = 4096$. The results are shown in Figure 2 with similar format as that of Figure 1. We can see, in composite model turbulence, when parallel diffusion sets in, the running perpendicular diffusion coefficient decreases, qualitatively as it did in the subdiffusive regime. However now it tends towards another constant having a lower level than either FLRW or the first standard perpendicular diffusion coefficient. True diffusion is recovered. We refer to this as "second diffusion." Note that the parallel diffusion is less than the standard quasilinear theory [5] result, presumably due to nonlinear effects.

DISCUSSION: SUBDIFFUSION AND SECOND DIFFUSION

Inherent in the QLT approach [5] is the assumption that the various types of transport, such as parallel and perpendicular scattering, can be computed independently. This assumption enters in QLT when the random forces are integrated along unperturbed trajectories. Without parallel scattering, if the field lines themselves randomize diffusively, the particles will also experience perpendicular diffusion. This can be explained that if the spread of field lines is at a rate of $D_\perp = \langle(\Delta x)^2\rangle/2\Delta z$ and particle speed is v, the (FLRW) diffusion of particles can be written as

$$\begin{aligned}\kappa_{xx} &= \frac{\left\langle(\Delta x)^2\right\rangle}{2\Delta t} \\ &= \frac{\Delta z}{\Delta t}\frac{\left\langle(\Delta x)^2\right\rangle}{2\Delta z} \\ &\sim vD_\perp. \end{aligned} \tag{2}$$

As long as the particles are free-streaming along field lines, the question as to whether their perpendicular transport becomes diffusive falls back on the issue of the complexity of the magnetic field viewed along field lines. Early on *Jokipii* [6] recognized that the structure of the magnetic field fluctuations perpendicular to the mean field might influence perpendicular transport, because the field lines sampled by a gyrating particle would tend to separate. In this way, *Jokipii* reasoned that FLRW perpendicular transport of particles would be accurate for low energy particles, which have gyroradii too small to sample very much transverse structure.

Skilling et al [15] carried out a calculation of cosmic ray scattering in the galaxy, noting apparently for the first time that divergence of neighboring field lines is crucial in situations in which particles' parallel scattering brings them back into the system from which they originate (in their case, scattering back into the galaxy). They noted that in order to scatter particles more than a gyroradius from their original field line, one needed to invoke field line separation. They used this idea to estimate, in effect, the distance a particle would scatter in the perpendicular direction for each unit of length transported along the magnetic field. The calculation of *Skilling et al.* is rather specialized in that it applies to a bounded inhomogeneous system (the galaxy) with parallel scattering centers located externally to the system; however, it is noteworthy that this is perhaps the first calculation in which it transpires that collisionless perpendicular transport depends explicitly upon the parallel transport.

It is not until somewhat later that it was recognized that the logical conclusion of the above reasoning is that in the limit of weak transverse structure, perpendicular transport is not diffusive. Urch [16] pointed out that particles' motions are not free streaming, but rather represent a parallel random walk, so that when particles back scatter through 90°, the perpendicular displacement is decreased. Accordingly, the estimate $\Delta z/\Delta t \sim v$ is incorrect, instead of, $\Delta z/\Delta t \approx (2\kappa_{zz}t)^{1/2}$, one finds instead that

$$\tilde{\kappa}_{xx} = D_{\perp}\sqrt{\frac{\kappa_{xx}}{\pi t}}. \quad (3)$$

(Kóta and Jokipii [8] revisited the issue of compound subdiffusion in the presence of strong parallel diffusion, and reached the same conclusion.) From this perspective, starting from weak transverse structure, what is needed is some effect to *restore* perpendicular diffusion.

The year following *Urch*, the same issue was again discussed by *Rechester and Rosenbluth* [14], but from a different perspective, namely that when particles' gyrocenters follow magnetic field lines, then the phase space density structure in directions perpendicular to the magnetic field evolves as an area preserving map. Field line wandering can make the phase space structure wrap up and fold, but without some additional effect, these surfaces cannot merge or break, so that in the presence of parallel scattering the entire process can be undone. The assertion (given essentially without proof) is that a small amount of scattering (in their case [14], presumably due to collisions) can cause the complex transverse phase space structure to blur slightly, so that parallel scattering can no longer cause the full restoration of the initial state. The conclusion [14] is that this slight additional collisional effect restores perpendicular transport to the FLRW rate, Eq. (2). These arguments are physically appealing but quantitatively inconsistent with our simulation results. There have been various calculations (e.g., [2]) that have adopted this assertion as proven, and, moreover, applicable to the collisionless limit. However to our knowledge our current line of research is the first to examine the issue of loss and restoration of perpendicular diffusion directly and quantitatively using accurate numerically determined particle orbits.

From our simulation research, examples of which are shown in the present paper, we conclude that for magnetic turbulence with little transverse structure subdiffusion is a long-lived state and mostly likely permanent state. But for magnetic turbulence with strong transverse structure, a regime of second diffusion is recovered. We view the "first" diffusion to be the evolution towards the FLRW limit. If there is no parallel scattering, this first diffusion limit is achieved (we will show evidence for this in a subsequent publication). Parallel scattering suppresses this tendency, decreasing the mean square transverse separation relative to the FLRW expectation. Subsequently one either gets subdiffusion, or if there is sufficient transverse structure, a new regime of second diffusions appears. This conclusion is qualitatively in accord with the physical reasoning of *Jokipii*, *Skilling et al*, and *Rechester and Rosenbluth*. However, none of these seems to have recognized that the (Coulomb) collisionless limit can result in a stable diffusion regime at a rate lower than FLRW, which is nevertheless manifestly nonlinear in a manner at least partially anticipated by these authors. We will present a more complete treatment of these effects in subsequent reports.

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