Lecture Note 1

1.1 Plasma

99% of the matter in the universe is in the plasma state.

Solid -> liquid -> Gas -> Plasma (The fourth state of matter)

Recall: Concept of Temperature
A gas in thermal equilibrium has particles of all velocities, and the most probable distribution of these velocities is known as the Maxwellian distribution.

The average kinetic energy E_{av} equals 1/2kT per degree of freedom. k is Boltzmann's constant.

Since T and E_{av} are so closed related, it is customary in plasma physics to give temperatures in units of energy. To avoid confusion on the number of dimensions involved, it is not E_{av}, but the energy corresponding to kT that is used to denote the temperature. For kT = 1 eV = 1.6 x 10^{-19} J, we have

$$T = \frac{1.6\times10^{-19}}{1.38\times10^{-23}} = 11,600$$

Thus the conversion factor is

1eV = 11,600 K

A plasma is a gas in which an important fraction of the atoms in ionized, so that the electrons and ions are separately free.

<u>When does this ionization occur? When the temperature is hot enough.</u>

Saha Equation:
Which tells us the amount of ionization to be expected in a gas in thermal equilibrium:

$$\frac{n_i}{n_n} \approx 2.4\times10^{21}\frac{T^{3/2}}{n_i}\exp(-U_i / kT)$$

Here n_i and n_n are, respectively, the number density (number per m^3) of ionized atoms and of neutral atoms, T is the gas temperature in K, k is Boltzmann's constant, and U_i is the ionization energy of the gas—that is, the number of ergs required to remove the outmost electron from an atom. For ordinary air at room temperature, T ~ 300 K, and U_i = 14.5 eV (for nitrogen). The fractional ionization $n_i/(n_i + n_n) \approx n_i / n_n$ is ridiculously low:

$$\frac{n_i}{n_n} \approx 10^{-122}$$

As the temperature is raised, the degree of ionization remains low until U_i is only a few times kT. Then n_i / n_n rises abruptly, and the gas is in plasma state.

Any ionized gas cannot be called a plasma, of course; there is always some small degree of ionization in any gas. A useful definition is as follows:

A plasma is a quasineutral gas of charged and neutral particles which exhibits collective behavior.

Collective behavior:

For ordinary air, the molecule is neutral, there is no net electromagnetic force on it, and the force of gravity is negligible. The molecule moves undisturbed until is makes collision with another molecule, and these collisions control the particle's motion. The situation is totally different in a plasma, which has charged particles. As these charges move around, they can generate local concentrations of positive or negative charge, which give rise to electric fields. Motion of charges also generates currents, and hence magnetic fields. These fields affect the motion of other charged particles far away.

It is the long-ranged Coulomb force that gives the plasma a large repertoire of possible motions and enriches the field of study known as plasma physics. By "collective behavior" we mean motions that depend not only on local conditions but on the state of the plasma in remote regions as well.

Quasi Neutral:

If a gas of electrons and ions (singly charged) has unequal numbers, there will be a net charge density, ρ.

$$\rho = n_e(-e) + n_i(+e) = e(n_i - n_e)$$

This will give rise to an electric field via.

$$\nabla \cdot E = \rho / \varepsilon_0 = e / \varepsilon_0 (n_i - n_e)$$

Example: Slab

$$\frac{dE}{dx} = \rho / \varepsilon_0$$
$$\Rightarrow E = \rho x / \varepsilon_0$$

This results in a force on the charges tending to expel whichever species is in excess. That is, if $n_i > n_e$, the E field causes n_i to decrease, n_e to increase tending to reduce the charge.

This restoring force is enormous!

Example

Consider $T_e = 1$ eV, $n_e = 10^{19}$ m^{-3} (a modest plasma; c.f. density of atmosphere $n_{molecules} \sim 3 \times 10^{25}$ m^{-3}

Suppose there is a small difference in ion & electron densities $\Delta n = (n_i - n_e)$

So $\rho = \Delta n \cdot e$

Then the force per unit volume at distance x is

$$F_e = \rho E = \rho^2 x / \varepsilon_0 = (\Delta n \cdot e)^2 x / \varepsilon_0$$

Take $\Delta n / n_e = 1\%, x = 0.10m$

$$F_e = (10^{17} \times 1.6 \times 10^{-19})^2 0.1 / 8.8 \times 10^{-12} = 3 \times 10^6 \text{ Nm}^{-3}$$

Compare this with the pressure force per unit volume ~P/x:

$P \sim n_e T_e$

$$F_p \sim 10^9 \times 1.6 \times 10^{-19} / 0.1 = 16 \text{ Nm}^{-3}$$

Electrostatic force >> Kinetic Pressure Force

This is one aspect of the fact that, because of being ionized, plasmas exhibit all sorts of collective behavior, different from neutral gasses, mediated by the long distance electromagnetic forces E, B

Another example (related) is that of longitudinal waves. In a normal gas, sound waves are propagated via the intermolecular action of collisions in a plasma, waves can propagate when collisions are negligible because of the coulomb interaction of the particles.

1.2 Debye Shielding

Plasma Density in Electrostatic Potential

When there is a varying potential, ϕ, the densities of electrons (and ions) is affected by it. If electrons are in thermal equilibrium, they will adopt a Boltzmann distribution of density

$$n_e \propto \exp(\frac{e\phi}{T_e})$$

This is because each electron, regardless of velocity possesses a potential energy $-e\phi$. Consequence is that

$$\begin{tikzcd}
\text{Electron/Ion Densities} \arrow[rr] & & \text{Electric Charge Density} \arrow[dl] \\
& \text{Electric Potential} \arrow[ul] &
\end{tikzcd}$$

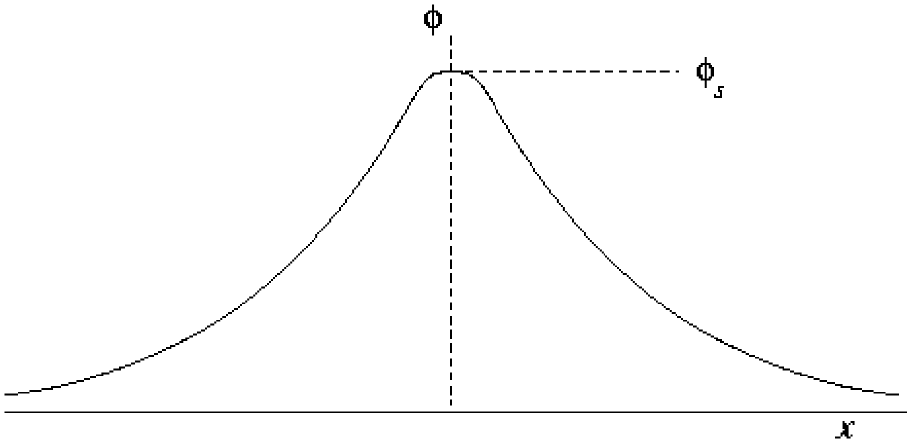
A self-consistent loop of dependencies occurs.

This is one elementary example of the general principle of plasmas requiring a self-consistent solution of Maxwell's equations of electrodynamics plus the particle dynamics of the plasma.

Debye Shielding

A slightly different approach to discussing quasi-neutrality leads to the important quantity called the Debye Length.

Suppose we put a plane grid into a plasma, held at a certain potential, ϕ_g.



Then, unlike the vacuum case, the perturbation to the potential falls off rather rapidly into the plasma. We can show this as follows. The important equations are Poisson's Equations:

$$\nabla^2 \phi = \frac{d^2\phi}{dx^2} = -\frac{e}{\varepsilon_0}(n_i - n_e)$$

Electron Density $n_e = n_\infty \exp(e\phi / T_e)$

[This is a Boltzmann factor; It assumes that electrons are in thermal equilibrium, n_∞ is density far from the grid (where we take $\phi = 0$)]

Ion Density $n_i = n_\infty$

[Applies far from grid by quasi-neutrality; we just assume, for the sake of this illustrative calculation that ion density is not perturbed by ϕ-perturbation.]

Substitute:

$$\frac{d^2\phi}{dx^2} = \frac{en_\infty}{\varepsilon_0}[\exp(\frac{e\phi}{T_e}) - 1]$$

This is a nasty nonlinear equations, but far from the grid $e\phi / T_e \ll 1$, so we can use a Taylor expansion: $\exp(e\phi / T_e) \simeq 1 + e\phi / T_e$

So

$$\frac{d^2\phi}{dx^2} = \frac{en_\infty}{\varepsilon_0}\frac{e}{T_e}\phi = \frac{e^2 n_\infty}{\varepsilon_0 T_e}\phi$$

Solutions:

$$\phi = \phi_0 \exp(-|x| / \lambda_D)$$
where
$$\lambda_D \equiv (\frac{\varepsilon_0 T_e}{e^2 n_\infty})^{1/2}$$
This is called the Debye Length.
Perturbations to the charge density and potential in a plasma tend to fall off with characteristic length λ_D.
In fusion plasma λ_D is typically small.
[e.g. $n_e = 10^{20} m^{-3}, T_e = 1keV, \lambda_D = 2\times10^{-5} m = 20\mu m$]

Usually we include as part of the definition of a plasma that $\lambda_D <<$ size of plasma. This ensures that collective effects, quasi-neutrality etc. are important. Otherwise they probably aren't.

1.3 Occurrence of plasma
Gas Discharge: Fluorescent Lights, spark gaps, arcs, welding, lighting
Controlled fusion
Ionosphere: Ionized belt surrounding earth
Interplanetary medium: Magnetospheres of planets, stars. Solar wind.
Stellar Astrophysics: Stars. Pulsars, Radiation-process.
Ion propulsion: Advanced space drives etc.
Space Technology : Interaction of spacecraft with environment
Gas lasers: Plasma discharge pumped lasers: CO_2, HeNe
Materials Processing: Surface treatment for hardening, Crystal growing.
Semiconductor Processing: Ion beam doping, plasma etching & sputtering.
Solid state plasmas: Behavior of semiconductors

1.4 The Plasma Parameter
Notice that in our development of Debye Shielding, we used $n_e e$ as the charge density and supposed that it could be taken as smooth and continuous. However if the density were so low that there were < 1 electron in the Debye Shielding region this approach would not be valid. Actually we have to address this problem in 3-D by defining the "Plasma Parameter", N_D, as
N_D = number of particles in the 'Debye Sphere'
$$= n \cdot \frac{4\pi}{3} \lambda_D{}^3 \propto \frac{T^{3/2}}{n^{1/2}}$$
If $N_D<1$ then the individual particle cannot be treated as a smooth continuum. It will be seen later that this means that collisions dominate the behavior: i.e. short range correlation is jus as important as the long range collective effects.
Often, therefore we add a further qualification of plasma:
$N_D >> 1$ (Collective effects dominate over collisions)

Summary
Plasma is an ionized gas in which collective effects dominate over collisions.
[$\lambda_D <<$ size, $N_D >> 1$]

Different Description of plasma
1. Single particle approach (Incomplete in itself)
2. Kinetic Theory (Boltzmann Equation)
3 Fluid Description Moments, Velocity, Pressure, Currents etc.
Uses of these.

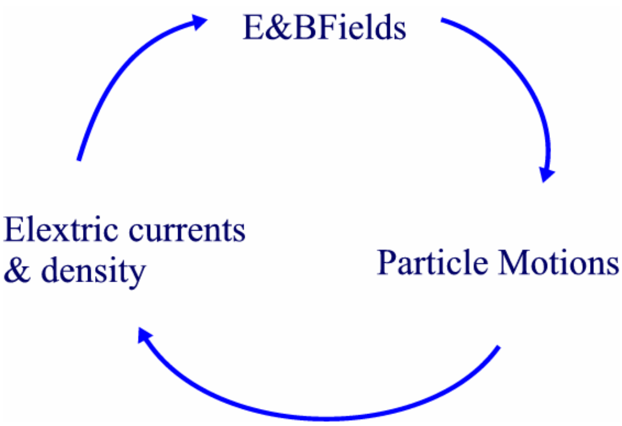
Single Particle Solutions $\rightarrow$ Orbits $\rightarrow$ Kinetic Theory Solutions $\rightarrow$ Transport Coefs. $\rightarrow$ Fluid Theory $\rightarrow$ Macroscopic Description.

All description should be consistent. Sometimes they are different ways of looking at same thing.

Equations of Plasma Physics: Maxwell Equations

Self Consistency

In solving plasma problems, one usually has a 'circular' system:



The problem is solved only when we have a model in which all parts are self consistent. We need a 'bootstrap' procedure.

Generally we have to do it in stages:
Calculate Plasma Response (to given E, B)
Get currents & charge densities
Calculate E & B for j, ρ
Then put it all together.