Plasma Frequency

The plasma frequency arises as a basic consequence of the restoring Coulomb interaction between oppositely charged particles. It is the plasma "ringing" response to a charge perturbation. We now consider the dynamic response of the plasma to an imposed charge separation. To study this, we displace electrons and ions and calculate the transient response. Since the electrons (-e) are much lighter than the ions, we assume the ions are immobile. Then

$$\begin{aligned} \mathbf{F} &= q\mathbf{E} \\ m_e \frac{d^2x}{dt^2} &= -e\frac{\rho x}{\varepsilon_0} \\ m_e \frac{d^2x}{dt^2} &= -\omega_{pe}^2 x \end{aligned}$$

which is the equation of a simple harmonic oscillator with electron plasma frequency:

$$\omega_{pe} = (\frac{e\rho}{m_e\varepsilon_0})^{1/2} = (\frac{ne^2}{m_e\varepsilon_0})^{1/2}$$

Note that the solution contains no wavenumber dependence (kx = 0), that is, x = x0 exp (iω_{ce}t) and the oscillation is not a propagating wave.

The ions oscillate much more slowly (by factor $\sqrt{m_e/m_i}$) about the centre of mass. At these frequencies, one can regard the ions as an irregular lattice of charge.

As a quick estimate, one can use

$$f_{pe} = 9\sqrt{n_e} \text{ Hz}$$

where n_e is in the unit of m^{-3}.

For example, for $n_e = 1 \times 10^{18} m^{-3}$(H-1NF), weh have $f_{pe} = 9$ GHz, well into the microwave region of the spectrum. For $n_e = 1 \times 10^{14} m^{-3}$ (a flame) we have $f_{pe} = 90$ MHz (FM radio)

It is also useful to not that the Debye length is a typical distance traversed by an electron with speed $u_{Te} = (kT_e/m_e)^{1/2}$, in a time ω_{pe}^{-1},

$$\lambda_D = (\frac{\varepsilon_0 kT_e}{n_e e^2})^{1/2} = \frac{u_{Te}}{\omega_{pe}}$$