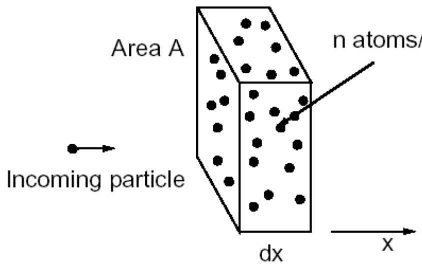
Collision Processes

Collisions mediate the transfer of energy and momentum between various species in a plasma, and as we shall see later, allow a treatment of highly ionized plasma as a single conducting fluid with resistivity determined by electron-ion collisions.

Collisions which conserve total kinetic energy are called elastic. Some examples are atom-atom, electron-atom, ion-atom (charge exchange) etc. In inelastic collisions, there is some exchange between potential and kinetic energies of the system. Examples are electron-impact ionization/excitation, collisions with surfaces etc. In this section, we consider both types of collision process, with particular emphasis on Coulomb collisions between charged particles, this being the dominant process in a plasma.

3.1 Mean free path and cross-section

To properly treat the physics of collisions we need to introduce the concept of mean free path - a measure of the likelihood of a collision event. Imagine electrons impinging on a box of neutral gas of cross-sectional area A.



If there are n_n atoms m^{-3} in volume element Adx , the total area of atoms in the volume (viewed along the x-axis) is $n_n A dx \sigma$ where σ is the cross-section and $n_n A dx \sigma \ll A$, so that there is no "shadowing". The fraction of particles making a collision is thus $n_n A dx \sigma / A$ - the fraction of the cross-section blocked by atoms. If Γ is the incident particle flux, the emerging flux is $\Gamma' = \Gamma(1 - n_n \sigma dx)$ and the change of Γ with distance is described by

$$\frac{d\Gamma}{dt} = -n_n \sigma \Gamma$$

The solution is

$$\Gamma = \Gamma_0 \exp(-x/\lambda_{mfp})$$

$\lambda_{mfp} = 1/n_n\sigma$, where λ_{mfp} is the mean free path for collisions characterised by the cross-section σ. The physics of the interaction is carried by σ, the rest is geometry. The mean free time between collisions, or collision time for particles of velocity v is $\tau = \lambda_{mfp}/v$ and the collision frequency is $\nu = \tau^{-1} = v/\lambda_{mfp} = n_n \sigma v$.

Averaging over all of the velocities in the distribution gives the average collision requency

$$\nu = n_n \overline{\sigma v}$$

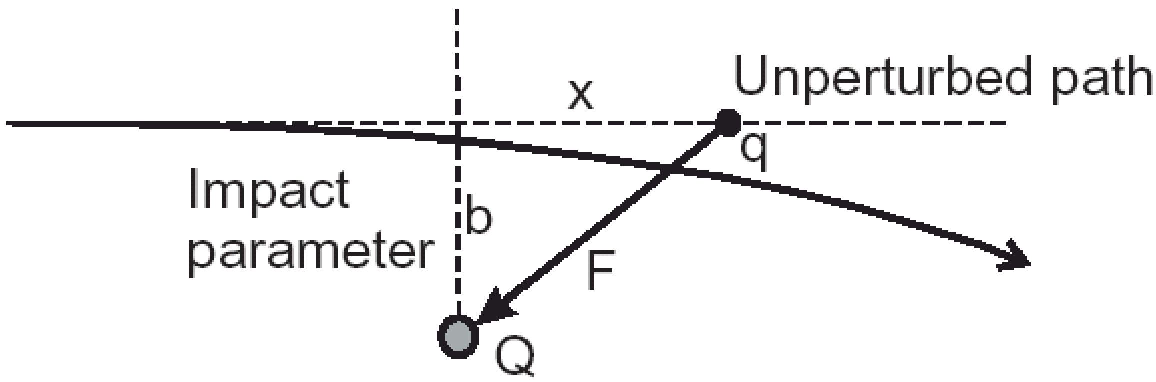
where we have allowed for the fact that σ can be energy dependent as we shall see below.

3.2 Coulomb collisions

Coulomb collisions between free particles in a plasma is an elastic process. Let us consider the Coulomb force between two test charges q and Q:

$$F = \frac{qQ}{4\pi\epsilon_0 r^2} \equiv \frac{C}{r^2}$$

This is a long range force and the cross-section for interaction of isolated charges is infinity! It is quite different from elastic "hard-sphere" encounters such as that which can occur between electrons and neutrals for example. In a plasma, however, the Debye shielding limits the range of the force so that an effective cross-section can be found. Nevertheless, because of the nature of the force, the most frequent Coulomb deflections result in only a small deviation of the particle path before it encounters another free charge. To produce an effective 90° scattering of the particle (and hence momentum transfer) requires an accumulation of many such glancing collisions. The collision cross-section is then calculated by the statistical analysis of many such small-angle encounters.



Consider the Coulomb force on an electron as it follows the unperturbed path shown in the picture, only the perpendicular component matters because the parallel compoent of the force reverses direction after q passes Q. Thus $F_\perp/F = b/r$ or

$$F_\perp = \frac{Cb}{r^3} = \frac{Cb}{(x^2 + b^2)^{3/2}}$$

where b is the impact parameter for the interaction. Since x (the parallel coordinate) changes with time, we integrate along the path to obtain the net perpendicular impulse delivered to q

$$\delta(mv_\perp) = \int_{-\infty}^{\infty} F_\perp dt = Cb \int_{-\infty}^{\infty} \frac{dx}{(x^2 + b^2)^{3/2}} \frac{dt}{dx}$$

Now

$$\int \frac{dx}{(x^2 + b^2)^{3/2}} = \frac{x}{b^2(x^2 + b^2)^{1/2}}$$

$$\rightarrow -1/b^2, x \rightarrow -\infty$$

$$\rightarrow 1/b^2, x \rightarrow \infty$$

so that

$$\delta v_{\perp} = \frac{2C}{mvb}$$

We consider a statistical average over a random distribution of such small angle collisions. For a random walk with step length δs, the total displacement after N steps is $\Delta s = \sqrt{N}\,\delta s$ (i.e. $(\Delta s)^2 = \sum(\delta s)^2$) where N is the number of steps. The total change in velocity is thus

$$(\Delta v_{\perp})^2 = N(\delta v_{\perp})^2$$

Now integrate over the range of impact parameters b to estimate the number of glancing collisions in time t. Small angle colisions are much less likely because of the geometrical effect

$$N(b) = n(2\pi b db)vt$$

We combine the three equations above and integrate over impact parameter:

$$(\Delta v_{\perp})^2 = \frac{8\pi n C^2 t}{m^2 v} \int_{b\,\min}^{b\,\max} \frac{db}{b}$$

b_{min} is the closest approach that satisfies the small deflection hypothesis. We obtain this be setting $\delta v_{\perp} = v$

$$b_{\min} = \frac{2C}{mv^2} = \frac{2qQ}{4\pi\epsilon_0 mv^2}$$

Outside λ_D the charge Q is not felt. We thus take $b_{max} = \lambda_D$ and

$$\int_{b\,\min}^{b\,\max} \frac{db}{b} = \ln(\frac{\lambda_D}{b_{\min}}) = \ln\Lambda$$

lnΛ is called the Coulomb logarithm and is a slowly varying function of electron density and temperature. For fusion plasma $\ln\Lambda \sim 6 - 16$. One usually sets $\ln\Lambda = 10$ in quantitative estimations.

We are finally in a position to find the elapsed time necessary for a net 90° deflection i.e. $(\Delta v_{\perp})2 = v^2$.

$$(\Delta v_{\perp})^2 = v^2 = \frac{8\pi n C^2 t}{m^2 v} \ln\Lambda$$

and

$$\nu_{90} = 1/t = \frac{8\pi n q^2 Q^2 \ln\Lambda}{16\pi^2\epsilon_0^2 m^2 v^3}$$

For electron-ion encounters, q = -e and Q = Ze so

$$\nu_{90ei} \equiv \nu_{ei} = \frac{n_i Z^2 e^4 \ln \Lambda}{2\pi^2 \epsilon_0^2 m^2 v^3}$$

and $\nu_{ei} = n_i \sigma_{ei} v$ implies that

$$\sigma_{ei} = \frac{Z^2 e^4 \ln \Lambda}{2\pi^2 \epsilon_0^2 m^2 v^4}$$

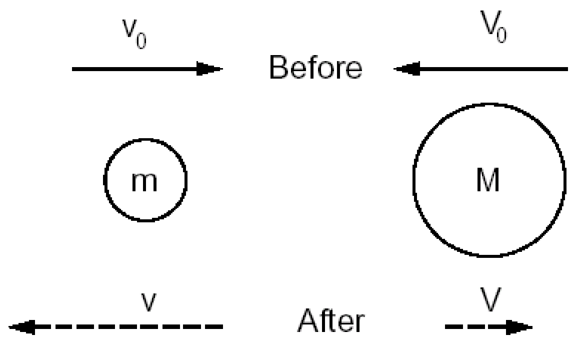
Let's review this derivation

- Perpendicular impulse $\sim 1/vb$
- Angular deflection $\Delta\theta \sim \delta v_\perp / v \sim 1/v^2$
- Random walk to scatter one radian ($\Delta v/v = 1$) $\sim 1/(\Delta\theta)^2 \rightarrow \sigma \sim 1/v^4$
- Integrate over b $\rightarrow \ln \Lambda$.

The dependence $\sigma \sim 1/v^4$ has very important ramifications. A high temperature plasma is essentially collisionless. This means that plasma resistance decreases as temperature increases. In some circumstances populations of particles can be continually accelerated, losing energy only through synchrotron radiation (for example runaway electrons in a tokamak).

3.3 Energy transfer in electron-ion collisions

A pervading theme in plasma physics is $m_e << m_i$. This has consequences for collisions between the species in that we expect very little energy transfer between the species. To illustrate this, consider such a collision in the centre-of-mass frame (a direct hit).



Collison between ion and electron in centre of mass frame.

$$mv_0 + MV_0 = 0 = mv + MV \quad \text{momentum}$$
$$mv_0^2 + MV_0^2 = mv^2 + MV^2 \quad \text{energy}$$

eliminate V, V_0:

$$mv_0^2 (1 + \frac{m}{M}) = mv^2 (1 + \frac{m}{M})$$

$$\Rightarrow \; \upsilon = \pm\upsilon_0 \text{ and } V = \pm V_0$$

Now translate to frame in which ion M is at rest (initially). In this frame theinitial electron energy is

$$E_{e0} = \frac{1}{2} m(\upsilon_0 - V_0)^2 = \frac{1}{2} m\upsilon_0^2 (1 + \frac{m}{M})^2$$

and the final ion energy is

$$E_i = \frac{1}{2} M(2V_0)^2 = 2M(\frac{m^2\upsilon_0^2}{M^2})$$

Their ratio is given by

$$\frac{\text{ion (final)}}{\text{electron (initial)}} = \frac{2m^2\upsilon_0^2}{\frac{1}{2} m\upsilon_0^2 M} (1 + \frac{m}{M})^2 \approx \frac{4m}{M}$$

and thus

$$e \to i, \quad \Delta E \sim \frac{4m_e E_e}{m_i}$$

$$e \to e, \quad \Delta E \sim E_e$$

$$i \to i, \quad \Delta E \sim E_i$$

For glancing collisions the energy transfer between ions and electrons is even less.

Coulomb collisions result in very poor energy transfer between electrons and ions. The rate of energy transfer is roughly (m_e/m_i) slower than the e-i collision frequency. On the other hand, energy transfer rate and collision frequency are the same for collisions between ions $\nu_{ii} \sim \nu_{ei}(m_e/m_i)$:

$$\frac{\nu_{ii}}{\nu_{ei}} = \frac{m_e^3\upsilon_e^3}{m_i^2\upsilon_i^3} = (\frac{m_e}{m_i})^{1/2} (\frac{m_e\upsilon_e^2}{m_i\upsilon_i^2})^{3/2} = (\frac{m_e}{m_i})^{1/2} \quad \text{for } T_e = T_i$$

That is,

$$\tau_E^{ei} \approx \frac{m_i}{m_e} \tau_{ei}$$

$$\tau_E^{ii} \approx \tau_{ii} \approx (\frac{m_i}{m_e})^{1/2} \tau_{ei}$$

$$\tau_E^{ee} \approx \tau_{ee} \approx \tau_{ei}$$