Plasma Physics Fall 2002

Problem Set 2

Due date: Monday, Sept. 30

1. Consider a charged particle moving in the x-y plane under the influence of a magnetic field varying slowly with x, pointing in the z-direction, $\mathbf{B} = \mathrm{B(x)}\ \mathbf{e}_z$ and uniform electric field in the y-direction, $\mathbf{E} = \mathrm{E}\ \mathbf{e}_y, (E/B < c)$. Show that as a result of the grad-B drift, the particle has work done on it by the electric field. Thus, show explicitly from conservation of energy, that the quantity $1/2mv^2/B$ is constant in this situation.

2. Electrons in a magnetic mirror have a distribution function $f_0(\mathbf{v})$ at its center, where the field magnitude is B_0. The highest field, in the mirror throat, is B_1. Calculate the fraction of electrons that is trapped if f_0 is

a) A maxwellian $f_0 = (\frac{m}{2\pi T})^{3/2} \exp(\frac{-mv^2}{2T})$

b) A squared Lorentrian $f_0 \propto \frac{1}{(1+v^2/v_t^2)^2}$

c) Proportional to $\frac{v_\perp}{v} \exp(\frac{-mv^2}{2T})$

3. A particle of mass m and charge q moves in uniform magnetic field B pointing in the z-direction under the influence of a line-charge of magnitude Q per unit length aligned alone the z-axis. (The configurations is invariant in the z-direction.)

a) Calculate the trajectory of its guiding center and the time elapsed before the x and y coordinates return to their initial values.

b) If the magnetic field is allowed to vary in time with a small constant time derivative dB/dt, calculate the evolution of the radial position of the guiding center ($r_g = \sqrt{x_g^2 + y_g^2}$) and of the particle kinetic energy.

c) Is the sum of particle kinetic energy and electrostatic potential energy ($q\phi$) constant? Do the charge and mass of the particle matter?