Plasma Physics Fall 2002
Problem Set 5

Due Date: Moday, Nov. 25

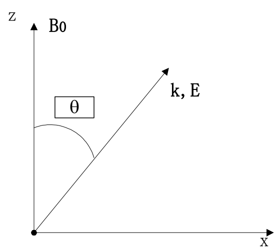
1. **a.** Compute the effect of collisional damping on the propagation of Langmuir waves (plasma oscillations), by adding a term $-nm\nu v$ to the electron equation of motion and rederiving the dispersion relation for $T_e = 0$

b. Write an explicit expression form $\mathrm{Im}(\omega)$ and show that its sign indicates that the waves is damped in time.

2. Find the dispersion relation for electrostatic electron waves propagating at an arbitrary angle θ relative to $\mathbf{B}_0$. Hint: Choose the x axis so that **k** and **E** lie in the x-z plane. (see Fig) Then

$$E_x = E_1 \sin\theta, E_z = E_1 \cos\theta, E_y = 0$$

and similarly for **k.** Solve the equations of motion and continuity and Poisson's equation in the usual way with n_0 uniform and $\mathbf{v}_0 = \mathbf{E}_0 = 0$.



a. Show that the answer is

$$\omega^2(\omega^2 - \omega_h^2) + \omega_c^2\omega_p^2\cos^2\theta = 0$$

b. Write out the two solutions of this quadratic for ω^2, and show that in the limits $\theta \to 0$ and $\theta \to \pi/2$, our previous results are recovered. Show that in these limits, one of the two solutions is a spurious root with no physical meaning.

c. By completing the square, show that the above equation is the equation of an ellipse:

$$\frac{(y-1)^2}{1^2} + \frac{x^2}{a^2} = 1$$

where $x = \cos\theta, y = 2\omega^2/\omega_h^2$, and $a = \omega_h^2/2\omega_c\omega_h$.

d. Plot the ellipse for $\omega_p/\omega_c = 1, 2, \text{and } \infty$.

e. Show that if $\omega_c > \omega_p$, the lower root for ω is always less than ω_p for any $\theta > 0$ and the upper root always lies between ω_c and ω_h; and that if $\omega_p > \omega_c$, the lower root lies below ω_c, while the upper root is between ω_p and ω_h.