Plasma Physics Fall 2002
Problem Set 6

Due Date: Monday, Dec. 16

1. A pulsar emits a broad spectrum of electromagnetic radiation, which is detected with a receiver tuned to the neighborhood of $f = 80$ MHz. Because of the dipersion in group velocity casused by the interstellar plasma, the observed frequency during each pulse drifts at a rate given by $df/dt = -5$ MHz/sec.

a. If the interstellar magnetic field is negligible and $\omega^2 >> \omega_p^2$, show that

$$\frac{df}{dt} \simeq -\frac{c}{x}\frac{f^3}{f_p^2}$$

where f_p is the plasma frequency and x is the distance of the pulsar.

b. If the average electron density in space is 2×10^5 m^{-3}, how away is the pulsar?

2. The R-Wave in the low-frequency region $\omega < \omega_c$ is called the whistler mode. Use the R-wave dispersion relationship

$$\frac{c^2k^2}{\omega^2} = 1 - \frac{\omega_p^2/\omega^2}{1 - \omega_c/\omega}$$

to show:

(a) The whistler mode has maximum phase velocity at $\omega = \omega_c$ and that this maxium is less than c (the velocity of light)

(b) The group velocity of the whistler mode is proportional to $\omega^{1/2}$.

3. The momentum equation for electrons in the presence of collisions with neutral particles is given by

$$mn\frac{d\mathbf{\upsilon}}{dt} = -en(\mathbf{E} + \mathbf{\upsilon} \times \mathbf{B}) - \nabla p - mn\nu_e(\mathbf{\upsilon} - \mathbf{u})$$

where ν_e is the momentum-exchange, large angle collision frequency. For

simplicity, assume that $u = 0$

a. Derive the linearized dispersion relationship of electromagnetic (transverse) waves in an unmagnetized plasma in the presense of collisions.

b. Assuming the ω is real and $k = k_{\text{Re}} + ik_{\text{Im}}$, from the dispersion relatioship deduce k_{Re}, the propagation wavenumber and $\delta = k_{\text{Im}}^{-1}$, the damping length (skin depth).

c. Find simplified expressions for δ in the limits (i) $\nu_e << \omega << \omega_{pe}$ (ii) $\omega << \nu_e << \omega_{pe}$

d. Calculate numerically the skin depth (in cm) in a plasma where $\omega/2\pi = f = 10^9$ Hz, $\nu_e = 3 \times 10^5$ sec^{-1}, and the density $n = 2 \times 10^{14}$ cm^{-3}.