Plasma Physics Fall 2002
Problem Set 7

Due Date: Monday, Jan. 6, 2003

1. *Vlasov Mush Transformation:* Discreteness can be eliminated by sub-dividing paricles indefinately while preserving the charge and mass. This sub-division can be described by the transformation,

$$q \rightarrow q/N$$
$$m \rightarrow m/N$$
$$n \rightarrow n/N$$

where N is the number of subdivisions, and q, m, and n, are particle charge, mass and density. Show that the Vlasov equation is invariant under this subdivision, while the discreteness parameter

$$\frac{1}{n\lambda_D^3}$$

is not. Furthermore show that,

$$\lim_{N\to\infty} \frac{1}{n\lambda_D^3} = 0$$

Thereby proving that the Vlasov equation becomes exact in the limit of zero discreteness.

2. *Bump-on-tail Instability:* Consider an electron distribution of the form,

$$f_e = \frac{(n_p + n_b)}{\pi^{3/2}v_e^3} \exp(-u^2/v_e^2) + \frac{n_b}{\pi^{3/2}v_b^3} \exp(-(u - u_b)^2/v_b^2)$$

with the orderings, $n_b << n_p, u_b >> v_e, v_e \sim v_b$. You may assume high frequency modes and neglect ion motion.

a. Make a plot of this distribution function and show where you expect unstable waves to occur.

b. What are the approximate ranges in frequency and wavenumber over which waves are unstable?

c. Calculate, for the fastest growing mode, the frequency, wavenumber and growth rate.

You can use the orderings given above and only need to express your answers to leading order in any small parameters.