

Space Plasma Physics Fall 2019

Problem Set 4

Due date: Dec. 14, 2019

1. What is ratio of the magnetic energy density ($B_1^2/2\mu_0$) to the kinetic energy density of the fluid motion ($\rho_{m0}V_1^2/2$) for the shear Alfvén wave? How about the same ratio for the magnetosonic (compressional Alfvén) wave?

2. Consider the propagation of Alfvén wave taking the displacement current into account. That is, start from the same equations as in the lectures but replace Ampère's law by

$$\nabla \times \mathbf{B} = \mu_0 \mathbf{J} + \frac{1}{c^2} \frac{\partial \mathbf{E}}{\partial t}$$

Derive the dispersion equation for the mode propagating perpendicular to the magnetic field into the form:

$$\frac{\omega^2}{k^2} = \frac{v_s^2 + v_A^2}{1 + v_A^2/c^2}$$

3. Under limits $V_A \gg V_s$ and $V_A \ll V_s$, (1) derive the phase speeds of fast and slow mode MHD waves, respectively; (2) Explain why a fast wave is a compressional Alfvén wave when $V_A \gg V_s$, and why it is a longitudinal wave when $V_A \ll V_s$