Space Plasma Physics Fall 2021

Problem Set 2

Due date: Nov. 26, 2021

1. MHD power generators may possibly be a more efficient way of converting heat into electricity. Think of one as consisting of a simple rectangular channel of (x-) width, a, (y-) height b, in which the plasma flows under pressure in the z-direction. Take the plasma density and velocity to be uniform. A uniform magnetic field, B, is applied in the y-direction and the walls a x = 0, a are electrodes where the electric current density (density j, assumed uniform) is picked off at a voltage difference Φ. Use the MHD equations to answer the following questions:

(a) If the resistivity, η, of the plasma is negligible, what is the plasma velocity?

(b) If the pressure is P_0 at z = 0, what is its value as a function of z?

(c) How much electric power is generated per unit length of the channel?

(d) What is the rate of doing work per unit channel length by the plasma pressure force?

(e) If η is not negligible but can be considered fixed, and the flow velocity and B-field are also fixed but the current density can be varied, what is the maximum electric power unit length that can be generated?

2. A θ - pinch (By symmetry, B has only z-component, j has only θ component and ∇p has only r component, so called because plasma currents flow in θ direction,) in MHD equilibrium has magnetic field that is

$$\mathrm{B(r)} = B_0 + (B_a - B_0)r/a$$

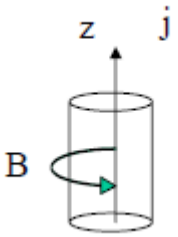
for $0 \ll \mathrm{r} \ll \mathrm{a}$

where the plasma edge is r = a, at which point the plasma pressure, p, is zero. Calculate:

(a) The pressure profile, p(r)
(b) The current density profile, j(r)
(c) The maximum possible value of the β, $2\mu_0 < p > /B_a^2$, where <p> is the volume average plasma pressure:

$$< \mathrm{p} > = \int_0^a p 2\pi r dr / \pi a^2$$

3. Z-pinch



So called because j follows in z-direction.
(a) For a z-pinch equilibrium which has zero plasma pressure at the plasma edge, r = a, prove by integrating the MHD force balance equation a second time that the volume-averaged pressure is a function only of the total current, and find the function.
(b) If a hydrogen plasma z-pinch has uniform density $\mathrm{n} = 10^{20}\ cm^{-3}$, temperature $T_e = T_i = T_0(1 - r^2/a^2)$ with $T_0 = 10\ keV$, and radius a = 0.01 m, what current is required?