Space Plasma Physics Fall 2021

Problem Set 3

Due date: Dec. 3, 2021

1. Sketch the magnetic field lines (with their orientations) in tow dimensions for $\boldsymbol{B} = (y, a^2x)$, where a is a parameter. Give the magnetic pressure force and the tension force. In which case is the current density null?

2. Show that a time dependent solution of the force-free ($\nabla \times \boldsymbol{B} = \alpha \boldsymbol{B}$) static equilibrium equation

$$\frac{\partial \boldsymbol{B}}{\partial t} = \frac{1}{\mu_0 \sigma} \nabla^2 \boldsymbol{B}$$

is given by

$$\boldsymbol{B} = \boldsymbol{B_0} e^{-t/\tau}$$

Where $\boldsymbol{B_0}$ is the solution of the vector Helmholtz equation

$$\nabla^2 B_0 + \alpha^2 \boldsymbol{B}_0 = 0$$

and $\tau = (\mu_0 \sigma)/\alpha^2$, with α = constant.

3. For a perfectly conducting fluid, one can show that

$$\frac{\mathrm{d}}{\mathrm{dt}}\left(\frac{\boldsymbol{B}}{\rho}\right) = \left(\frac{\boldsymbol{B}}{\rho} \cdot \nabla\right) \boldsymbol{V}$$

where ρ is the fluid density and V is the fluid velocity. Use this relation to establish that in a perfectly conducting fluid, the fluid elements that lie initially on a magnetic flux line continue to lie on a flux line.

4. If the magnetic field in the solar wind is "frozen" in the plasma, show that the magnetic field near the solar equatorial plane take on the shape of an Archimedian spiral

$$\phi = \frac{\omega}{V_{sw}} r + \phi_0$$

Where Φ is an azimuthal angle viewed from the polar axis, ω is the angular rotational velocity of the Sun, r is the distance from the Sun, and V_{sw} is the solar wind velocity. At the orbit of the earth (1AU), the solar wind speed is about 400 km/s, what is the angle between the field and the line drawn from the Sun to an observer at 1AU?

5. Assuming that the polar cap has a radius of 15° of latitude or 1,500 km, plasma flow speeds of 330 m/s, and an average magnetic field of 0.55 gauss, estimate the total potential drop across the polar cap. What is the potential drop over a distance equal to the diameter of the tail (50 Re) in the undisturbed solar wind, assuming the solar wind speed is 400 km/s and the interplanetary magnetic field (IMF) pointing

southward with a magnitude of 5 nT. How much percentage of the IMF magnetic field that would impact the geomagnetic cross section of the magnetosphere reconnects with the geomagnetic field?