Space Plasma Physics Fall 2021

Problem Set 4

Due date: Dec. 10, 2021

1. What is ratio of the magnetic energy density ($B_1^2/2\mu_0$) to the kinetic energy density of the fluid motion ($\rho_{m0}V_1^2/2$) for the shear Alfven wave? How about the same ratio for the magnetosonic (compressional Alfven) wave?

2. Consider the propagation of Alfven wave taking the displacement current into account. That is, start from the same equations as in the lectures but replace Ampere's law by

$$\nabla \times \mathbf{B} = \mu_0 \boldsymbol{J} + \frac{1}{c^2}\frac{\partial \boldsymbol{E}}{\partial t}$$

Derive the dispersion equation for the mode propagating perpendicular to the magnetic field into the form:

$$\frac{\omega^2}{k^2} = \frac{v_s^2 + v_A^2}{1 + v_A^2/c^2}$$

3. Under limits $V_A \gg V_s$ and $V_A \ll V_s$, (1) derive the phase speeds of fast and slow mode MHD waves, respectively; (2) Explain why a fast wave is a compressional Alfven wave when $V_A \gg V_s$, and why it is a longitudinal wave when $V_A \ll V_s$

4. Show that the plasma pressure and magnetic pressure fluctuations reinforce one another in the fast MHD wave, whereas the fluctuations oppose one another in the slow MHD wave.