IOP Publishing — Plasma Physics and Controlled Fusion

Plasma Phys. Control. Fusion **50** (2008) 062001 (6pp) doi:10.1088/0741-3335/50/6/062001

BRIEF COMMUNICATION

Modeling solar energetic particle by a relativistic kappa-type distribution

Fuliang Xiao[1,2], Qinghua Zhou[1], Chunxian Li[1] and Aijun Cai[1]

[1] School of Physics and Electronic Sciences, Changsha University of Science and Technology, Changsha 410076, People's Republic of China

[2] State Key Laboratory of Space Weather, Chinese Academy of Sciences, Beijing 100080, People's Republic of China

Received 27 January 2008, in final form 12 March 2008
Published 7 April 2008
Online at stacks.iop.org/PPCF/50/062001

Abstract

A recently developed relativistic kappa-type (KT) is adopted to model the observed spectra of solar energetic protons. The KT distribution is found to fit well with the observed data in the energies of ∼1–100 MeV and 100–1000 MeV, suggesting that the solar energetic proton flux follows the power law at both the lower energies and the relativistic energies, and the KT distribution may present a further physical insight into those space plasmas where energetic particles exist.

Energetic particles (e.g. solar energetic particles (SEP)) found in planetary magnetospheres and other plasmas often exhibit a pronounced non-Maxwellian high-energy tail distribution that can be well modeled by a generalized Lorentzian (kappa) distribution (e.g. [1–6]). Analogous to the big effect of the solar wind on the Earth's magnetosphere [7–11], SEP associated with the solar flare, coronal and interplanetary shocks have a great impact on the space weather [12–15]. It is therefore important to be capable of specifying and even forecasting the fluxes of those high-energy SEP. The typical kappa distribution is widely adopted in previous studies (e.g. [16–18]) to model the behavior of high energetic particles. Various observational data have shown that energetic particles can be well characterized by a power-law spectrum, including those data at the geostationary orbit during the November 1993 geomagnetic storm [19], the electron measurements from the LANL satellites and from GOES 10 [20] and the energy spectra of ions accelerated in impulsive and gradual solar events [21]. However, the kappa distribution $\propto v^{-2(\kappa+1)}$ instead of $\propto p^{-(\kappa+1)}$ at the relativistic energy (where κ, v and p are the spectral index, velocity and momentum of particles). This appears to be in conflict with the power law since the relativistic energy $\propto p$ instead of $\propto v^2$. Recently, Xiao [22] developed a fully relativistic kappa-type (KT) distribution to model the highly energetic particles in a more physically realistic way in plasmas where magnetic mirror geometries occur. The KT distribution is found to incorporate features of the well-known KT and loss-cone type, and satisfies the power law both at the lower energies and the relativistic energies. Xiao *et al* [23] further utilize the new KT distribution to fit the energetic electrons' spectrum observed by the SOPA instrument on board the 1989-046 and LANL-01A satellites at the geosynchronous orbit

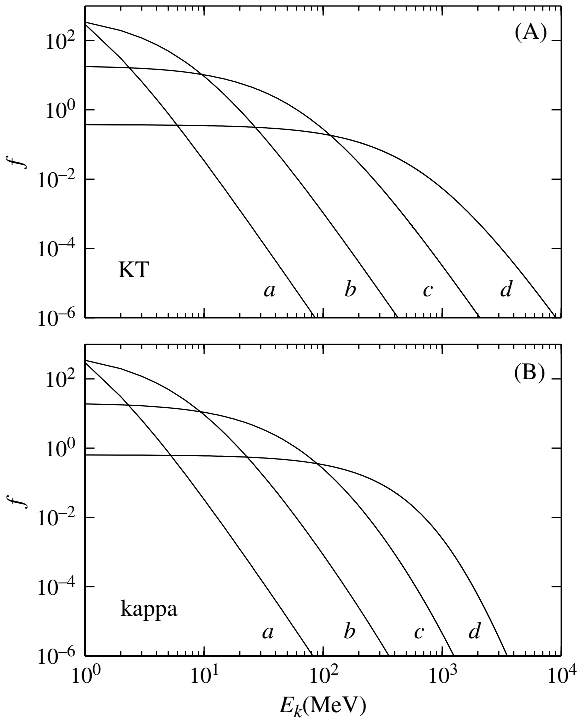

Figure 1. (*A*) Curves of the KT distribution for $\kappa = 4$ and indicated values of $\theta^2 = 0.0002$ (a), 0.002 (b), 0.02 (c) and 0.2 (d). Since θ^2 is scaled by m_0c^2 (i.e. ~940 MeV), θ^2 approximately corresponds to 0.188 MeV, 1.88 MeV, 18.8 MeV, 188 MeV, respectively. (*B*) Same except for kappa distribution.

and find that the new KT distribution fits well with the observed data during different universal times in both lower and higher energies. In order to investigate whether the power-law spectra are ubiquitous in space plasmas, we therefore adopt the KT distribution to model SEP spectra observed by the IMP 8 and Helios 1 and 2 spacecraft [21] and also in the October 28 2003 solar event [24].

The typical relation between the differential flux $j(E)$ and the distribution function $f(p)$ can be written [25] as

$$j(E) = p^2 f(p), \tag{1}$$

where E is the kinetic energy of particles. In the relativistic limit, we have

$$p^2 = \frac{m_0 E(E + 2E_0)}{E_0}, \tag{2}$$

where $E_0(= m_0c^2)$ is the rest mass energy of particles with $c(= 3 \times 10^{10}\,\mathrm{cm\,s^{-1}})$ being the speed of light.

Using such scaled variables: $p_s = p/m_0c$ and $E_s = E/E_0$, and considering that the total number density N is the same in the cases of the unscaled variables and the scaled variables, we can obtain the following relation [23]:

$$j = \frac{cE_s(E_s + 2)}{E_0} f(p_s). \tag{3}$$

Equation (3), which relates the differential flux and the distribution function in the scaled variables, presents the primary scheme to model the observed data.

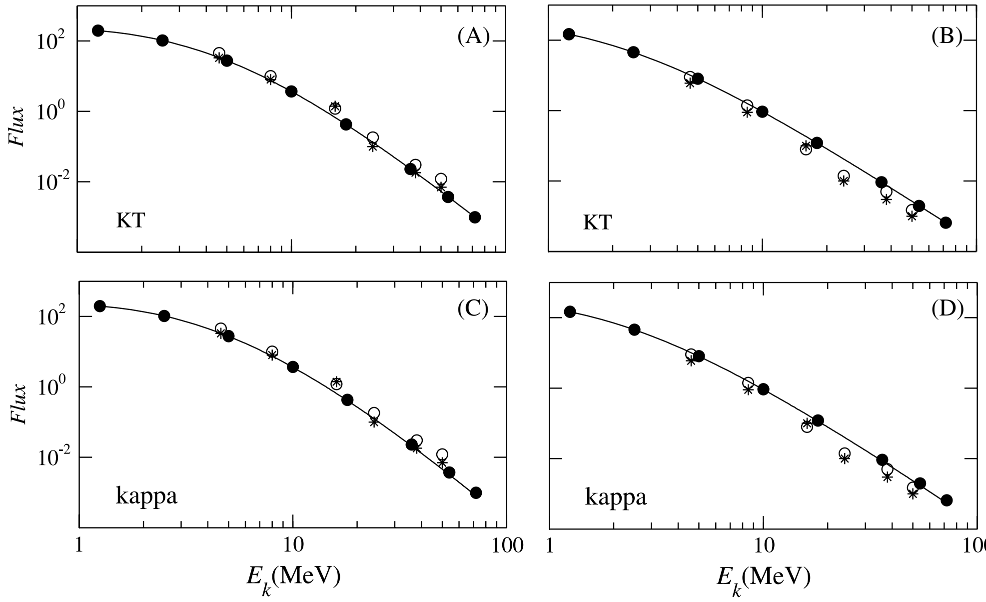

Figure 2. (upper) Fitting curves (——) of the differential flux (in units of (cm^2 s sr kev)$^{-1}$) by the KT distribution for energetic protons observed during the September 23 1978 (*A*) and the March 1 1979 (*B*) SEP events by Helios 1 (⋆) , Helios 2 (○) and IMP 8 (●) spacecraft. (lower) Same except for kappa distribution. (lower) Same except by the kappa distribution during the 1978 September 23 (*C*) and the 1979 March 1 (*D*) SEP events by Helios 1 (⋆), Helios 2 (○), and IMP 8 (•) spacecraft.

The general KT distribution in the scaled variables is introduced in [22]:

$$f(p_s) = \frac{N}{4\pi I}\left[1 + \frac{\sqrt{1+p_s^2} - 1}{\kappa\theta^2}\right]^{-(\kappa+1)}, \tag{4}$$

where I is the normalized constant given by

$$I = \frac{8B(3/2, \kappa-2)}{2\kappa-1}\Big\{3F\big(\kappa+1, 5/2; \kappa+1/2; 1-2/\kappa\theta^2\big) \\ + (\kappa-2)F\big(\kappa+1, 3/2; \kappa+1/2; 1-2/\kappa\theta^2\big)\Big\}, \tag{5}$$

where Γ, B and F are the gamma function, the beta function and the hypergeometric function, respectively, θ^2 is the thermal characteristic parameter scaled by m_0c^2 and N is the number density of particles. The KT distribution is found to have a different influence from the regular kappa distribution on the whistler-mode instability [26].

Based on the regular non-relativistic kappa distribution function adopted by previous authors [16, 17], we shall use the following kappa distribution in the scaled variables [27]:

$$f^{\kappa}(p_s) = \frac{\Gamma(\kappa+1)}{\pi^{3/2}\theta_\kappa^3\kappa^{3/2}\Gamma(\kappa-1/2)}\left[1 + \frac{p_s^2}{\kappa\theta_\kappa^2}\right]^{-(\kappa+1)}; \tag{6}$$

here θ_κ^2 denotes the thermal characteristic parameters (scaled by m_0c^2). The regular kappa distribution function (6) with a typical value of κ in the range 2–6 is found to provide a good representation for the high energy tail population of the space plasmas. Since $\sqrt{1+p_s^2} - 1 \approx p_s^2/2$ in (4) in the non-relativistic limit, we assume $\theta_\kappa^2 = 2\theta^2$ in the following calculation in order to make a consistent and direct comparison.

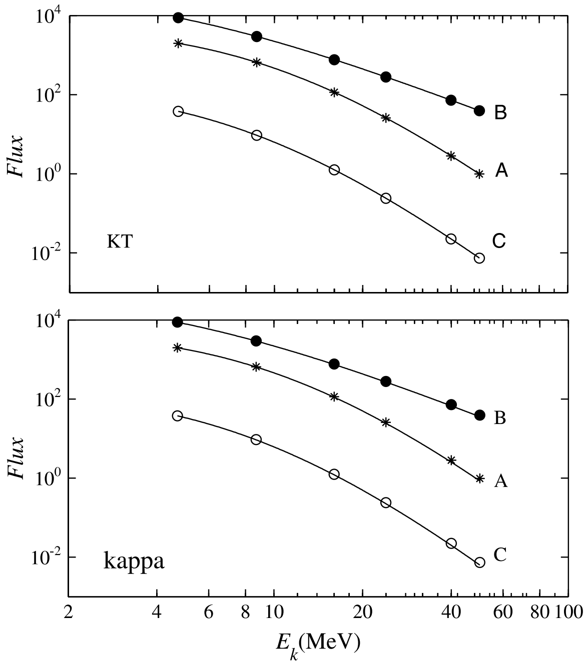

Figure 3. (upper) Fitting curves (——) of the differential flux by KT distribution with the data (evaluated from figure 6 in [21]) for different times (A–C) during the November 14 1980 SEP event. (lower) Same except for kappa distribution.

In figure 1, we show examples of the KT (4) and kappa (6) distribution behavior of energetic protons for different indicated values of θ^2 and κ by setting $N = 1$. As θ^2 increases, the KT distribution is found to decrease more slowly than the kappa distribution, in particular above the energies ~100 MeV. There is a turning point when θ^2 is larger (e.g. $\theta^2 \gtrsim 18$ MeV); the distribution varies slowly before the turning point and then decreases sharply after it. This feature suggests that the energetic electrons' spectra do not follow a simple power law but depend on the electron energy range [28], and the KT distribution obeys an approximate power law at the relativistic energies.

Following the work [29], Reames *et al* [21] have presented further an extensive investigation on the historic database provided by the IMP 8 and Helios 1 and 2 spacecraft to search for the origin of spatial and temporal invariance in the large SEP events. By using equations (3)–(6) and the least-square fitting method, in figure 2, we present the fitting curves (solid line) of the differential flux for energetic protons observed during the September 23 1978 and the March 1 1979 SEP events by Helios 1, Helios 2 and IMP 8 spacecraft. These data are evaluated from figures 1 and 2 (time interval B) in [21]. It is found that both the KT and kappa distributions give an adequate fit to these data in all the energies 1–100 MeV with the following best fitting parameters: during the September 23 1978 event, $\kappa = 5$, $\theta^2 = 8 \times 10^{-4}$ (or 0.752 MeV), $N = 5 \times 10^{-6}\,\text{cm}^{-2}$ and during the March 1 1979 event, $\kappa = 4$, $\theta^2 = 4 \times 10^{-4}$ (or 0.376 MeV), $N = 5 \times 10^{-6}\,\text{cm}^{-2}$.

Simulated results (solid line) of the differential flux with observed data for Helios 1 during the different times (A)–(C) of the November 14 1980 SEP event are shown in figure 3. Similarly, both the KT and the kappa distributions fit well with those data in the energies 4–60 MeV with the following best fitting parameters: time A, $\kappa = 6$, $\theta^2 = 1.7 \times 10^{-3}$ (or 1.598 MeV), $N = 1.2 \times 10^{-4}\,\text{cm}^{-2}$; time B, $\kappa = 3$, $\theta^2 = 1.0 \times 10^{-3}$ (or 0.94 MeV),

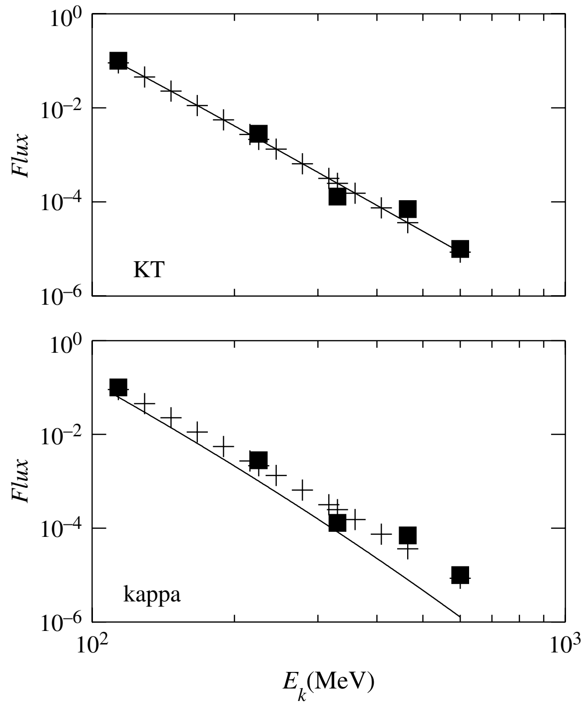

Figure 4. (upper) Fitting curves (——) of the differential flux by KT distribution with the data (evaluated from figure 6 in [24]) for the October 28 2003 solar event. Cross symbols denote the data of balloon flights over Apatity and black square symbols denote the data from GOES-10/11 spacecraft. (lower) Same except for kappa distribution.

$N = 8\times10^{-4}\,\text{cm}^{-2}$; and time C, $\kappa = 6, \theta^2 = 1.3\times10^{-3}$ (or 1.222 MeV), $N = 3\times10^{-6}\,\text{cm}^{-2}$. These results indicate that solar energetic protons with energies ∼100 MeV and below can be best characterized by both the KT and the kappa distributions, at least in the cases of interest.

A flare occurring on October 28 2003, which produced a relativistic particle event at Earth [24], provides a very convenient approach to examine the spectra behavior of the SEP with energies above 100 MeV. In figure 4 we provide simulations (solid line) of the relativistic protons during the October 28 2003 solar event. The KT distribution gives an adequate fit to those data in all the energies 100–600 MeV with the best fitting parameters as follows: $\kappa = 6, \theta^2 = 1.85\times10^{-3}$ (or 1.739 MeV) and $N = 5\times10^{-4}\,\text{cm}^{-2}$. The kappa distribution is found to fit well with the data at energies below ∼200 MeV but start deviating away from the data at higher energies, indicating that the typical kappa distribution perhaps does not satisfy an approximate power law at high energies, particularly $\gtrsim$300 MeV.

In summary, based on the previous papers [21, 24], we have presented simulations of the SEP spectra observed by the IMP 8 and Helios 1 and 2 spacecraft, and also in the October 28 2003 solar event by using a fully relativistic KT distribution function, and found an adequate fit to the observed data at both energies below and above 100 MeV, indicating that the KT distribution obeys an approximate power law at the relativistic energies. Since a previous work has shown that the energetic particle spectra are characterized by a power law which depends on the particle energy range, the current result suggests that the KT distribution may be a useful tool to model the highly energetic particles in those space plasma environments where highly energetic particles are present, but this needs to be further confirmed.

Acknowledgment

This work was supported by the National Natural Science Foundation of China Grant 40774078 and the Visiting Scholar Foundation of State Key Laboratory of Space Weather, CAS.

References

[1] Vasyliunas V M 1968 *J. Geophys. Res.* **73** 2839
[2] Christon S P *et al* 1988 *J. Geophys. Res.* **93** 2562
[3] Summers D and Thorne R M 1991 *Phys. Fluids* B **3** 1835
[4] Summers D and Thorne R M 1992 *J. Geophys. Res.* **97** 16827
[5] Maksimovic M, Pierrard V and Riley P 1997 *Geophys. Res. Lett.* **24** 1151
[6] Viñas A F, Mace R L and Benson R F 2005 *J. Geophys. Res.* **110** 6202
[7] Li X *et al* 2001 *Geophys. Res. Lett.* **28** 1887
[8] Li X *et al* 2005 *Space Weather* **3** S04001
[9] Zong Q-G and Wilken B 1998 *Geophys. Res. Lett.* **25** 4121
[10] Zong Q-G *et al* 2001 *J. Geophys. Res.* **106** 25541
[11] Zong Q-G *et al* 2007 *Geophys. Res. Lett.* **34** L12105
[12] Leske R A, Mewaldt R A and Stone E C 2001 *J. Geophys. Res.* **106** 30011
[13] Hudson M K *et al* 2004 *J. Atmos. Sol. Terr. Phys.* **66** 1389
[14] Kress B K *et al* 2004 *Geophys. Res. Lett.* **31** L04808
[15] Smith Z K, Murtagh W and Smithtro C 2004 *J. Geophys. Res.* **109** A01110
[16] Dasso S, Gratton F T and Farrugia C J 2003 *J. Geophys. Res.* **108** 1149
[17] Saito S *et al* 2000 *Ann. Geophys.* **18** 1216
[18] Maksimovic M, Pierrard V and Lemaire J F 1997 *Astron. Astrophys.* **324** 725
[19] Freeman J W *et al* 1998 *J. Geophys. Res.* **103** 26
Freeman J W *et al* 1998 *J. Geophys. Res.* **103** 251
[20] Burin des Roziers E and Li X 2006 *Space Weather* **4** S06007
[21] Reames D V *et al* 1997 *Astrophys. J.* **484** 515
[22] Xiao F 2006 *Plasma Phys. Control. Fusion* **48** 203
[23] Xiao F *et al* 2008 *J. Geophys. Res.* **113** at press
[24] Miroshnichenko L I *et al* 2005 *J. Geophys. Res.* **110** A09S08
[25] Schulz M and Lanzerotti L J 1974 *Particle Diffusion in the Radiation Belts Phys. Chem. Space* vol 7 (New York: Springer)
[26] Xiao F *et al* 2006 *Plasma Phys. Control. Fusion* **48** 1437
[27] Xiao F, Thorne R M and Summers D 1998 *Phys. Plasmas* **5** 2489
[28] Summers D and Ma C 2000 *J. Geophys. Res.* **105** 2625
[29] Reames D V, Barbier L M and Ng C K 1996 *Astrophys. J.* **466** 473