Solar wind-magnetosphere energy coupling function fitting: Results from a global MHD simulation

C. Wang1, J. P. Han1, H. Li1, Z. Peng1, and J. D. Richardson3

1State Key Laboratory of Space Weather, Center for Space Science and Applied Research, Chinese Academy of Sciences, Beijing, China, 2College of Earth Sciences, University of Chinese Academy of Sciences, Beijing, China, 3Kavli Institute for Astrophysics and Space Research, Massachusetts Institute of Technology, Cambridge, Massachusetts, USA

Abstract Quantitatively estimating the energy input from the solar wind into the magnetosphere on a global scale is still an observational challenge. We perform three-dimensional magnetohydrodynamic (MHD) simulations to derive the energy coupling function. Based on 240 numerical real runs, the energy coupling function is given by

$$E_n = 3.78 \times 10^3 n_{sw}^{0.24} V_{sw}^{0.84} B_{T}^{0.06} \sin^2(\theta/2) + 0.25$$

We study the correlations between the energy coupling function and a wide variety of magnetospheric activity, such as the indices of Dst, Kp, ap, AE, AU, AL, the polar cap index, and the hemispheric auroral power. The results indicate that this energy coupling function gives better correlations than the $\epsilon$ function. This result is also applied to a storm event under northward interplanetary magnetic field conditions. About 13% of the solar wind kinetic energy is transferred into the magnetosphere and about 35% of the input energy is dissipated in the ionosphere, consistent with previous studies.

1. Introduction

The processes of energy transmission, conversion, and dissipation in the solar wind-magnetosphere-ionosphere (SW-M-I) coupling system are an important fundamental question in space physics. The energy input into the magnetosphere from the solar wind drives many space weather phenomena, such as magnetic storms, substorms, aurora, and other magnetospheric activities [Akasofu, 1981] and also is the ultimate source of the dynamics of the magnetosphere-ionosphere (M-I) system. With the growing interest in space weather activities during the past decade, the study of the magnetospheric energy budget has drawn more and more attention. Quantitative estimation of the energy input and dissipation in the magnetosphere and ionosphere system have become one of the key challenges of the solar-terrestrial physics and space weather [e.g., Lu et al., 1998; Turner et al., 2001; Pulkkinen et al., 2002; Østgaard et al., 2002a; Tanskanen et al., 2002; Steren, 1984, and references therein].

The question of how much solar wind energy is transferred into the magnetosphere has been extensively studied since the early 1960s. As direct observation cannot determine the energy input on a global scale, a large number of solar wind coupling functions have been developed which rely on the use of proxies to estimate the energy entering into the Earth's magnetosphere [e.g., Perreault and Akasofu, 1978; Akasofu, 1981; Vasyliunas et al., 1982]. Some of these coupling functions have emerged as key space physics parameters (for a review, see, e.g., Gonzalez [1990], Newell et al. [2007], and Finch and Lockwood [2007]).

It is natural to relate the energy input to the solar wind conditions and interplanetary magnetic field (IMF). The mass density ($\rho$), velocity ($V$), and hence the dynamic pressure ($P_d$) of the solar wind were the earliest solar wind parameters considered, even before the solar wind was discovered to be continuous and not merely episodic [Chapman and Ferrara, 1930]. Some early work suggested that the solar wind velocity might be an important factor determining the solar wind energy input into the magnetosphere [Crooker et al., 1977]. But more recently, Crooker and Gringauz [1993] and Papitashvili et al. [2000] argue that the solar wind velocity does not work as well as the earlier studies suggest. Dungey [1961] investigated the importance of the $Z$ component of the IMF, $B_z$, and dayside magnetic field merging, finding that the IMF $B_z$ better predicts the energy input than the solar wind velocity and the dynamic pressure. However, the IMF $B_z$ predicts only a little better than a quarter of the observed variance in the magnetospheric state variables [Newell et al., 2007]. Various combinations of the basic solar wind parameters have been tried in subsequent decades [e.g., Kan and Lee, 1979; Wygant et al., 1983; Scoury and Russell, 1991; Temerin and Li, 2006; Newell et al., 2007]. Most of the coupling functions are proxies which only describe the energy input indirectly and
do not give the magnitude of the energy input [e.g., Gonzalez and Mozer, 1974; Østgaard et al., 2002a, and references therein].

Quantitative derivations of the energy transferred to the magnetosphere have also been conducted. Perreault and Akasofu [1978] investigated the energy budget during magnetic storms. The energy input is parameterized in the form of the $\varepsilon$ parameter as follows:

$$\varepsilon = \frac{4 \pi}{\mu_0} V B^2 l_0^2 \sin^4 \left( \frac{\theta}{2} \right)$$  

(1)

The parameters are in SI units. $V$ is the solar wind velocity, $B$ is the IMF magnitude, $\theta$ is the IMF clock angle with $\tan \theta = B_Y / B_Z$, and $l_0$ is an empirical scaling factor denoting the linear dimension of the “effective cross-sectional area” of the solar wind-magnetosphere (SW-M) interaction, which is usually assumed to be $7 R_E$ (Earth radii). The $\varepsilon$ parameter is based on the Poynting flux, which represents the flow of electromagnetic energy from the solar wind into the magnetosphere. Akasofu [1981] considered it as a first approximation for the SW-M energy coupling function. Although the $\varepsilon$ parameter was originally scaled to geomagnetic storm events, it has been used to estimate the energy input in a wide range of time scales and conditions. The $\varepsilon$ parameter has become one of the most widely used energy coupling functions. However, this form was derived empirically, and thus, the physical interpretation is not clear [Koskinen and Tanskanen, 2002]. The scaling factor $l_0 = 7 R_E$ was obtained by assuming that the energy input equals the estimated energy dissipation, which mainly included the Joule heating and auroral particles precipitation in the ionosphere, and the ring current dissipation. Its value has significant uncertainties since the estimates of the energy dissipation via the ionosphere, the ring current, and the plasmoids have been revised [Palmroth et al., 2003]. Koskinen and Tanskanen [2002] also suggested that a scaling parameter of $1.5–2$ should be applied to $l_0$ to account for some substorm-related tail energy sinks, such as the plasma sheet heating and the energy carried by plasmoids in the magnetotail.

Another energy coupling function, $P$, derived by Vasyliunas et al. [1982] using dimensional analysis, is a physics-based estimate of the power extracted from the solar wind. The function depends on the assumptions about the energy coupling mechanisms, specifically on the relative importance of electromagnetic coupling (MHD flow effects), ionospheric conductivity effects (through Birkeland currents), and viscous coupling. With the dimension of power, $P$ has a general expression given by

$$P = \rho V^3 l_{CF}^2 F \left( M_A^2, H, R, \theta \right)$$  

(2)

$$M_A^2 = \frac{\mu_0 \rho V^2}{B_T^2}$$  

(3)

$$R = \frac{V l_{CF}}{V}$$  

(4)

$$H = \mu_0 \Sigma_p V$$  

(5)

$$l_{CF} = \left( \frac{M_A^2}{\mu_0 \rho V^2} \right)^{1/6}$$  

(6)

where $\rho$ is the solar wind mass density, $V$ is the solar wind velocity, $l_{CF}$ is the Chapman-Ferraro magnetopause distance, $F$ is an unspecified dimensionless function of the dimensionless ratios ($M_A^2, H, R$) and of the IMF clock angle $\theta$, $M_A$ is the Alfvén-Mach number based on the transverse part of the magnetic field $\left( B_T = \sqrt{B_Y^2 + B_Z^2} \right)$. $H$ measures the relative importance of ionospheric conductivity compared to inertial effects in determining the strength of Birkeland currents [Hill and Rassbach, 1975], $R$ is the Reynolds number, $\mu_0 = 4 \pi \times 10^{-7}$ H m$^{-1}$ is the magnetic permeability of free space, $\nu$ is the effective kinematic viscosity, and $M_E = 8.06 \times 10^{22}$ A m$^{-2}$ is the Earth’s magnetic dipole moment. However, the energy coupling function proposed by Vasyliunas et al. [1982] is a general function with some undetermined parameters. According to the general expression, many other energy coupling functions are introduced based on different databases or methods [e.g., Murayama, 1982; Bargatze et al., 1986; Xu and Shi, 1986; Stamper et al., 1999; Finch and
However, most of these energy coupling functions do not quantitatively provide the energy input from the solar wind because of the undetermined coefficients [Murayama, 1982; Bargatze et al., 1986; Stamper et al., 1999; Finch and Lockwood, 2007]. Most recently, Tenfjord and Ostgaard [2013] employed the P parameter approach to derive two new dynamic energy coupling functions for both geomagnetic storms and longer time series using 13 years of OMNI and SuperMag data.

Global MHD simulations provide an effective approach to investigate the global energy flow into the SW-M-I system [Papadopoulos et al., 1999]. Palmroth et al. [2003] simulated the energy flow from the solar wind into the magnetosphere during a major magnetic storm using a global 3-D MHD model, the Grand Unified Magnetosphere Ionosphere Coupling Simulation. They studied the energy transfer distribution and found that during the storm’s main phase, the energy was transferred from the plane parallel and antiparallel to the IMF clock angle. In addition, they compared the simulation result with the empirical ε parameter and found that the ε parameter was about 4 times smaller than simulation result. Subsequently, a few other studies of energy input and dissipation in the magnetosphere have been performed using global MHD simulation codes [Palmroth et al., 2005, 2006, 2010, 2012; Pulkitinen et al., 2002, 2008, 2010]. Recently, Lu et al. [2013] studied the energy input distribution for northward and southward IMFs using the global MHD modeling framework Space Weather Modeling Framework. They conclude that most of the energy flux inflow occurs near the polar cusps on the magnetopause for northward IMF, the electromagnetic energy input mainly occurs at the tail lobe behind the cusps, and the mechanical energy input mainly occurs at the near-equatorial dayside magnetopause for southward IMF. Global MHD simulations have not previously been used to establish the qualitative relationship between the energy input and the solar wind parameters; that relationship is the main topic of this paper.

We will focus on the energy transferred from the solar wind into the magnetosphere based on a global MHD simulation model and derive an energy coupling function from numerical simulation data. The paper is organized as follows. Section 2 describes the simulation model and data sets. Section 3 introduces the methodology of the magnetopause identification and the determination of the energy coupling function. Section 4 presents the energy coupling function fit results and a correlation analysis between our energy input results and geomagnetic indices. Sections 5 and 6 give the discussion and summary, respectively.

2. Simulation Model and Data Sets

The global 3-D Piecewise Parabolic Method with a Lagrangian Remap (PPMLR) MHD simulation model developed by Hu et al. [2005, 2007] is adopted in this study to simulate the SW-M-I coupling system. This code has been used successfully to model the interaction of interplanetary shocks with the magnetosphere, large-scale current systems, and Kelvin-Helmholtz instabilities at the magnetopause [Wang et al., 2013]. This numerical scheme is of third-order spatial precision and second-order temporal precision, with very small numerical dissipation. The GSM coordinate system is used, and the computational domain extends from $X = -300 R_E$ to $X = 30 R_E$ along the Sun-Earth line and from $-150 R_E$ to $150 R_E$ in the Y and Z directions. The whole domain is represented by a $160 \times 162 \times 162$ grid. In the inner domain of $|X, Y, Z| \leq 10 R_E$, a uniform mesh is laid out with a grid spacing of 0.4 $R_E$. The grid spacing outside increases according to a geometrical series of common ratio 1.05 along each axis. The inner boundary is set to be 3 $R_E$ in order to avoid the complexity associated with the plasmasphere and strong magnetic field near Earth’s surface. For simplicity, a uniform Pedersen conductance of 5 S is assumed in the ionosphere and the Hall conductance is set to 0. The code solves the MHD equations in fully conservative form in the SW-M system. Meanwhile, electrostatic equations are solved in the ionosphere. The coupling between the magnetosphere and the ionosphere consists of a mapping of field-aligned current from the inner boundary of the magnetosphere to the ionosphere and of a mapping of the electric potential in the opposite direction. Both mappings are along the Earth’s dipole field lines. The conservative form of the MHD equations guarantees that the mass, momentum, and energy are conserved in the numerical simulation. Other details about the global MHD model can be found in Hu et al. [2007].

In this study, a total of 240 numerical test runs with different solar wind conditions are conducted to investigate the quantitative relationship between the energy input and the solar wind parameters. The solar wind velocity is varied from 400 to 800 km/s, the solar wind number density from 5 to 20 cm$^{-3}$, the IMF $B_x$ from $-5$ nT to $-20$ nT, and the IMF $B_y$ from $-10$ nT to 10 nT in 5 nT intervals. The detailed combinations of the...
Table 1. The Combination of Solar Wind Conditions for 240 Simulation Cases

<table>
<thead>
<tr>
<th>Density (cm$^{-3}$)</th>
<th>Velocity (km/s)</th>
<th>IMF $B_x$ (nT)</th>
<th>IMF $B_y$ (nT)</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>400</td>
<td>−5</td>
<td>−10</td>
</tr>
<tr>
<td>10</td>
<td>600</td>
<td>−10</td>
<td>−5</td>
</tr>
<tr>
<td>15</td>
<td>800</td>
<td>−15</td>
<td>0</td>
</tr>
<tr>
<td>20</td>
<td>−20</td>
<td>5</td>
<td>10</td>
</tr>
</tbody>
</table>

Solar wind conditions are listed in Table 1. Furthermore, 37 additional numerical tests are also carried out to study the effect of IMF clock angle $\theta_v$ changing from 0° to 360° with the interval of 10° and other solar wind conditions remaining the typical values (400 km/s, −5 nT, 5 cm$^{-3}$). For each set of solar wind conditions, the computation continues until a steady state has persisted for more than 5 h in physical time.

3. Methodology

3.1. Identifying the Magnetopause Surface From MHD Simulation Data

Evaluation of the solar wind energy transfer to the magnetosphere requires a definition of an appropriate surface, i.e., the magnetopause. An automatic identification of the magnetopause surface is of great help for a significant number of numerical simulation data sets corresponding to different solar wind conditions. Palmroth et al. [2003] developed a method to determine the magnetopause by finding approximately the inner edge of the void encompassed by the solar wind streamlines. We follow their approach with a minor improvement.

First, a set of streamlines is created at $X = +25 R_E$, well beyond the bow shock. The streamlines grid is set in the $YZ$ plane in a circle with a radius of $25 R_E$. The $X$ axis is at the center of the circle. The radial distance between neighboring streamlines is $0.5 R_E$ and the angular separation is $1^\circ$, giving 18,000 streamlines in total.

Second, the set of 18,000 streamlines is mapped in steps of $0.5 R_E$ in the $−X$ direction. For each step, the algorithm searches for a void of streamlines starting from the $X$ axis, where the streamlines started to bend around the magnetosphere. The subsolar point of the magnetopause is where the void becomes larger than $0.5 R_E$. We divided the $YZ$ plane into $3^\circ$ sectors in order to search for the inner boundary. In each sector the streamlines are sorted by their distance $\sqrt{Y^2 + Z^2}$ from the $X$ axis. As pointed out by Palmroth et al. [2003], some streamlines may enter the magnetosphere and affect the accuracy of the magnetopause location. Comparing with the plasma density contours, the three closest streamlines are excluded and the magnetopause is defined to be the fourth closest streamline. The search is carried out until $X = −60 R_E$.

The magnetopause determined from this method under northward and southward IMF are shown in Figures 1 and 2, respectively. Detailed investigation and comparison of the magnetopause under different IMFs are not the purposes of this paper. We also compared the magnetopause determined by the streamline method with the contour plot of plasma density in the equatorial and meridian planes and found they are highly consistent.

3.2. Estimating the Energy Flow Through the Magnetopause in the MHD Simulation

Once the magnetopause surface is identified, the energy flow across the magnetopause surface can be calculated [Palmroth et al., 2003]. The area and the normal vector (the positive direction points outward from the magnetopause) of each surface element are found. Then the energy flux across each quadrangular surface element ($dE_q$) is calculated as

$$dE_q = dA \mathbf{K} \cdot \mathbf{n}$$  \hspace{1cm} (7)

where $dA$ is the area of the surface element and $\mathbf{n}$ is the unit normal vector. $\mathbf{K}$ is the total energy flux defined as

$$\mathbf{K} = \left( U + \frac{B^2}{2\mu_0} \right) \mathbf{v} + \frac{1}{\mu_0} \mathbf{E} \times \mathbf{B}$$  \hspace{1cm} (8)

where $U = \frac{\gamma p}{\gamma - 1} + \frac{1}{2} \rho v^2 + \frac{B^2}{2\mu_0}$ is the total energy density, including the thermal energy density, kinetic energy density, and magnetic energy density, $\gamma = 5/3$ is the polytropic exponent, $p$ is the thermal pressure, $\mathbf{B}$ is the magnetic field, $\mathbf{v}$ is the velocity, and $\mathbf{E} = -\mathbf{v} \times \mathbf{B}$ is the convection electric field. The total energy flux $\mathbf{K}$ is
interpolated from the PPMLR-MHD simulation at the center of each surface element with the vertices of the surface element. The total energy flux through the surface is then the integration of the energy fluxes for each surface element,

\[ E_{in} = \int dE_q \]  

(9)

3.3. Dimensional Analysis of Energy Coupling Function

After the energy input is calculated, we can fit the energy coupling function of the SW-M coupling system based on the dimensional analysis made by Vasyliunas et al. [1982]. The dimensional analysis indicates that the upstream energy flux density, including electromagnetic energy flux density and mechanical energy flux density, would be an appropriate input function. In equation (2), once the unspecified dimensionless function \( F \) is determined, the energy input would be obtained. If the coupling mechanism involves only MHD flows, i.e., ionospheric conductance and finite gyroradius effects (including viscosity) can be neglected, \( F \) is a function of \( M_A^2 \) and \( \theta \) alone; using the explicit expressions for \( I_C \) and \( M_A \) and assuming that the dependence of \( F \) on \( M_A^2 \) can be approximated by a power law

\[ F(M_A^2, \theta) = M_A^{-2\alpha}G(\theta) \]  

(10)

The formula can then be rewritten as

\[ P = C_1 \mu_0^{-1/3-\alpha} M_E^{2/3} \rho^{2/3-\alpha} V^{7/3-2\alpha} B_T^{2\alpha} G(\theta) [W] \]  

(11)

where \( C_1 \) is the fit coefficient. In the equation, \( \rho \) is mass density in kg/m\(^3\), \( V \) is velocity in m/s, and \( B_T \) is IMF magnitude in T. \( G(\theta) \) is a function of the IMF clock angle in the form of \( G(\theta) = \sin^\theta \left( \frac{\theta}{2} \right) + \delta \) and \( \beta \) and \( \delta \)
Figure 2. (top) The 3-D magnetopause under the southward IMF. The determined magnetopause with the contour plot of plasma density in (left bottom) the equatorial plane and (right bottom) the meridian plane are given.

are free parameters to be determined from 37 quasi-steady state test runs in which \( \theta \) was varied from 0° to 360°. The index \( \alpha \) was determined by a nonlinear fitting technique based on 240 simulation runs. Thus, an empirical formula for the energy transfer from the solar wind to the magnetosphere has been obtained.

If the correlation coefficient between the calculated energy transfer from energy coupling function \( P \) and the computed value from MHD simulation is high enough, then the assumption that \( F \) depends only on \( M_A \) and \( \theta \) but not on \( H \) or \( R \) is justified.

4. Results

4.1. Nonlinear Fitting of the Energy Coupling Function

Based on the dimensional analysis proposed by Vasyliunas et al. [1982], an approximation of the energy transfer from the solar wind into the magnetosphere can be obtained from equation (11). Once \( G(\theta) \) and \( \alpha \) are determined, the energy transfer power will be obtained quantitatively.

We calculate the energy transfer as described above for all 37 cases and did a nonlinear fit to \( G(\theta) \). The black line in Figure 3 shows the variation of the total energy input from equation (8) versus the IMF clock angle. The blue line denotes the electromagnetic energy input, and the green line denotes the sum of the kinetic energy and the thermal energy input. The red line represents a nonlinear fit to the total energy input which has an \( R^2 = 0.98 \), indicating an excellent fit. The prediction efficiency (PE), which describes how close the fit values are to the simulation values and is defined as \( PE = 1 - \frac{(E_{\text{fit}} - E_{\text{sim}})^2}{\sigma_{\text{sim}}^2} \), is equal to 1 if the fit values are equal to the simulation values. The PE of the \( G(\theta) \) fit is about 0.98, which indicates that the nonlinear fit is nearly perfect. The fit expression for \( G(\theta) \) is

\[
G(\theta) = \sin^2 \left( \frac{\theta}{2} \right) + 0.25
\]

(12)
Figure 3. The energy input obtained from equation (8) versus the IMF clock angle. The black solid line is the total energy input from the solar wind into the magnetosphere under various IMF clock angles. The blue solid line is the electromagnetic energy input. The green solid line is the sum of the kinetic energy input and the thermal energy input. The red line is the result of \( G(\theta) \), which is fitted by the total energy input. The IMF clock angle is from 0 to 360 with other solar wind parameters unchanged. The \( R^2 \) represents the goodness of nonlinear fitting, which is equal to 1 for the perfect fitting.

We performed test runs with different solar wind velocities, densities, and total IMF intensities. These runs may give different energy inputs, but they do not significantly affect the dependence of the energy input on the IMF clock angle. Figure 4 shows the energy input normalized by the mean value of the energy input for IMF clock angles from 0° to 360°. The black dashed line is the normalized \( G(\theta) \) from equation (12), the red solid line is the normalized energy input from the simulation for IMF = 5 nT, the green solid line is for IMF = 10 nT, and the blue solid line is for IMF = 20 nT. The results for different solar wind velocities and number densities are similar. Therefore, it is clear that different solar wind conditions do not influence the \( G(\theta) \) fit results.

After fitting \( G(\theta) \), the index \( \alpha \) was determined to be 0.43 from the nonlinear fit to 240 simulation cases with \( R^2 = 0.98 \). The combination of solar wind conditions can be found in Table 1. Thus, the energy input \( E_{in} \) is

\[
E_{in} = 3.78 \times 10^7 n^{0.24} V^{1.47} B_T^{0.86} \left[ \sin^2 \frac{\theta}{2} + 0.25 \right] \text{[W]}
\]

This formula gives the energy input \( E_{in} \) in watts, with the solar wind number density \( n \) in \( \text{cm}^{-3} \), the solar wind velocity \( V \) in \( \text{km/s} \), and the transverse magnetic field \( B_T = \sqrt{B_x^2 + B_y^2} \) in nT.

Figure 5 compares the energy input calculated from our energy coupling function (black dots) with those calculated from the \( \varepsilon \) function of equation (1) (red dots). The horizontal axis denotes the energy input identified from the MHD simulations. The lines are a linear fit to the results. The red dashed line is where the simulation and coupling function are equal. The correlation coefficients for the \( \varepsilon \) function and our result are 0.86 and 0.99, respectively. The energy input from our energy coupling function matches the simulation results very well. The distribution of the points from our energy coupling function are more concentrated than that for the \( \varepsilon \) parameter. The \( \varepsilon \) function underestimates the energy input by a factor of 4 to 5 and does not include the energy dissipated by plasma sheet heating and plasmoid outflow, which is similar to the
Even considering only the energy dissipated in the inner magnetosphere, the $\epsilon$ parameter underestimates the energy input by a factor of 1 to 2 relative to the simulation results. The prediction efficiency (PE) for our function (0.97) is larger than that for the $\epsilon$ function (0.50), indicating that the nonlinear fit works well.

From equation (13), the power law exponent of the solar wind velocity is larger than the exponents of other solar wind parameters. Moreover, the energy coupling function also shows high dependence on the IMF clock angle with a power law exponent of 2.70 for $\sin(\theta/2)$. This is very close to the result of $8/3$ obtained by Newell et al. [2007] and is consistent with the studies of Gosling et al. [1986, 1990] with ISEE 2 data sets. This implies that the energy coupling function is more sensitive to the variation of the IMF clock angle and the solar wind velocity than the variation of other solar wind parameters and the IMF magnitude. Note that the energy input includes two parts, similar to the result of Newell et al. [2008], a power sine function of the IMF clock angle and a constant independent of the IMF clock angle. Newell et al. [2008] examined different energy coupling function combinations with one part representing the merging term and the other part representing the viscous term to find which combination best predicts magnetospheric activity. This method is different from other studies in which the energy coupling function is a power law of $\sin(\theta/2)$. The first part of this energy coupling function is the electromagnetic energy input, which is controlled by the IMF clock angle involving the dayside magnetic reconnection. The second part is the energy transfer caused by other processes, such as high-latitude magnetic reconnection and viscous effect, which is basically independent of the IMF clock angle. The mechanical energy input is dominant under northward IMF conditions and is about 15–20% of the total energy input under southward IMF conditions.

### 4.2. Correlations Between Magnetospheric Activity Indices and Energy Coupling Functions

To validate the energy coupling function obtained from the simulation data, we calculate the correlations between magnetospheric activity variables and energy coupling functions following Newell et al. [2007]. The indices we use are the Auroral Electrojet (AE), a measure of the overall intensity of auroral currents; $AU$, which measures currents in the dayside auroral oval; $AL$, which is a measure of currents in the nightside auroral oval; Dst, a measure of ring current strength [Newell et al., 2007]; polar cap index (PC), a proxy for the amount of energy transferred from the solar wind into the magnetosphere through direct driving (i.e., merging), which measures the level of geomagnetic disturbance at Thule station in the polar cap due to ionospheric Hall currents and distant field-aligned currents at the poleward edge of the auroral oval [Troschichev et al., 1988] with 1 h resolution; $Kp$, which measures general planetary wide geomagnetic disturbances at midlatitude; and the $ap$ index, with 3 h resolution [Akasofu, 1981; Burton et al., 1975; Newell et al., 2007].

<table>
<thead>
<tr>
<th>Rank, $f$</th>
<th>AE</th>
<th>Dst</th>
<th>PC</th>
<th>$AU$</th>
<th>$AL$</th>
<th>$Kp$</th>
<th>$ap$</th>
<th>Aurora Power</th>
<th>$\sqrt{\sum r^2/n}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>This study</td>
<td>0.65</td>
<td>−0.53</td>
<td>0.69</td>
<td>0.57</td>
<td>−0.63</td>
<td>0.64</td>
<td>0.74</td>
<td>0.66</td>
<td>0.64</td>
</tr>
<tr>
<td>$\epsilon$</td>
<td>0.42</td>
<td>−0.42</td>
<td>0.47</td>
<td>0.33</td>
<td>−0.43</td>
<td>0.39</td>
<td>0.59</td>
<td>0.47</td>
<td>0.45</td>
</tr>
</tbody>
</table>

The energy coupling functions from this study perform better than the ($\epsilon$) function in terms of the mean correlation coefficient.

Figure 5. Comparison between the energy coupling functions result of 240 cases and the calculation results from simulations. The red dashed line is where the simulation and coupling function are equal. The black dots are the results from the energy coupling function from this study. The red dots are the results from the $\epsilon$ function.
Figure 6. Relationships between the aurora power, Dst, AE index, and the energy inputs from two energy coupling functions. The AE and Dst index data cover from 1966 to 2012. The hemispheric auroral power data are provided by the Total Energy Detector (TED) instrument on board the NOAA/POES. The $\epsilon$ function correlates worse with AE and Dst index and aurora power than that for the energy coupling function from this study.

magnetopause current contribution is removed from the Dst index used here, $\text{Dst}^* = \text{Dst} - 7.26 \sqrt{P_d} + 11$ [O’Brien and McPherron, 2000]. These indices are obtained from the NASA OMNI database (http://omniweb.gsfc.nasa.gov/ow.html). In addition, the hemispheric auroral power data are also studied, which represent substorm activity and are provided by the Total Energy Detector (TED) instrument on board the NOAA/POES (formerly TIROS) series of polar orbiting satellites. These indices cover the time period from 1966 to 2012. We use all available OMNI data, which have been time shifted to the nose of the bow shock. Most previous coupling functions, except the $\epsilon$ parameter, do not give the energy input quantitatively, so we do the correlation analysis for our energy coupling function and $\epsilon$ function.

The results of correlations between the energy coupling functions and magnetospheric activity variables are listed in Table 2. The energy coupling function from this study ($E_{in}$) correlates better with these eight variables than the $\epsilon$ function, with a mean correlation coefficient of 0.64. For example, the scatterplots of AE, Dst index, and aurora power versus energy input are shown in Figure 6. The correlation coefficients between the AE index and $\epsilon$ and $E_{in}$ are 0.42 and 0.65, respectively. The correlation coefficients are $-0.42$ and $-0.53$ for the $P_d$-corrected Dst index and 0.47 and 0.66 for aurora power. The latter (and higher) values are for our index $E_{in}$. Considering the amount of data, the correlation coefficients indicate that our energy coupling function is linear correlated with the geomagnetic indices. Overall, the energy coupling function from this study does a better job than the $\epsilon$ function in terms of the predicting magnetospheric activity.

5. Discussion
5.1. Error Analysis
In general, the uncertainties or errors in determining the energy input using the MHD approach are mainly due to features of the MHD model itself, such as the model precision, the grid resolution, and the magnetopause determination. The PPMLR-MHD model in this study has a formal accuracy of the third order in space and the second order in time with low numerical dissipation. The grid resolution in this study is $0.4 R_E$ in $[X, Y, Z]$ $\leq 10 R_E$ region. To test the effect of the grid resolution on the energy input determination, we increase the grid resolution from 0.4 $R_E$ to 0.2 $R_E$. The results indicate that the energy input differences of the two different grid resolutions are less than 3%. As indicated by Janhunen et al. [2012], the grid resolution will change the energy input but the effect on the results is small. We have improved the streamline method used to determine the magnetopause by increasing the number of streamlines and the resolution compared to Palmroth et al. [2003]. In addition, we calculated what the energy input would be if the magnetopause location were in error by 0.5 $R_E$ or $-0.5 R_E$ and found a 6% difference. Even if the magnetopause...
is not determined very precisely, the estimate of the energy input does not change significantly. The MHD simulation model has other limitations, such as the lack of the accurate ring current model and an oversimplified ionospheric conductance model. In Figure 6, the correlation for the $Dst$ index is poorer than that for the other indices. This poorer correlation is probably due to the absence of an accurate ring current model coupled to the global MHD simulation model. In addition, the ionospheric conductance may have effects on the energy input. The uniform Pedersen conductance used in our model is not realistic. However, we have conducted a sensitivity analysis, varying the ionospheric conductance for the same solar wind conditions. We chose conductances of 1 S, 3 S, 7 S, and 10 S for this study. Changing the conductance over this range changes the energy input by only 8%. Non-MHD effects, such as nonthermal particles and non-MHD waves, may also affect the energy input. However, the effects of non-MHD processes on the energy input are thought to be weak compared with MHD processes.

5.2. Comparison With Previous Studies

With the accumulation of observational data, several energy coupling functions have been derived using solar wind and IMF conditions with various criteria for event selection. Based on the dimensional analysis conducted by Vasyliunas et al. [1982] and observations, Finch and Lockwood [2007] analyzed the dependence of the correlation coefficient on the timescale $T$ of the events used and the value of $\alpha$ in the energy coupling function and found that the optimum $\alpha$ is 0.3 for timescales exceeding 28 days. At $T = 3$ h, there is a slight increase in $\alpha$, to about 0.4. Murayama [1982] found $\alpha = 0.4$ for $T$ near 1 day. Bargatze et al. [1986] found $\alpha = 0.5$ for $T < 1$ h and Stamper et al. [1999] found $\alpha = 0.38$ for $T = 1$ year. The list of the $\alpha$ values is shown in Table 3. Similarly, different studies obtain different exponents of $\sin(\theta/2)$. Perreault and Akasofu [1978] and Scurry and Russell [1991] both find that the exponent is 4. However, Vasyliunas et al. [1982] finds 2, Temerin and Li [2006] gives 6, and Newell et al. [2007] finds 8/3 as shown in Table 3. Our fit result is 2.70, very close to the result of Newell et al. [2007] and consistent with the studies of Gosling et al. [1986, 1990] with ISEE 2 data sets. In addition, Olsson et al. [2004] calculated the ionospheric Joule heating with the Poynting flux method based on the observational data and found that the parameter $\rho V^3$ correlated with the ionospheric Joule heating better than the $\varepsilon$ parameter.

5.3. Scope of Application

According to the dimensional analysis made by Vasyliunas et al. [1982], the IMF clock angle only affects the $G(\theta)$ determination. We found $G(\theta)$ from the 37 quasi-steady state test runs with the IMF clock angle $\theta$ changing from $0^\circ$ to $360^\circ$. Therefore, the energy coupling function $\alpha$ takes into account the influence of the IMF clock angle. To further validate this claim, we carried out test runs of more than one hundred cases with $B_z > 0$. We calculated the energy input of these test runs using the energy coupling function and compared the calculated values with the simulation values. The result is similar to that in Figure 5, with the scatter points distributed near the line of equality. Therefore, we conclude that the IMF clock angle only affects the $G(\theta)$ determination and does not significantly affect the $\alpha$ determination, so the energy coupling function can be applied to both southward and northward IMF conditions.

5.4. An Application Example

As an application example, we use our new derived energy coupling function to reexamine the energy budget issue during the major magnetic storm event on 21–22 January 2005, which has been studied by Du et al. [2008] by using the $\varepsilon$ function. During 21–22 January 2005, a fast shock arrived at 1712 UT resulting in the decrease of $SYM-H$ with a peak around $-41$ nT at 1847 UT. The sudden increased dynamic pressure caused by the fast shock led to the storm initial phase. During the storm main phase, the IMF $B_z$ turned northward from 1946 UT (the first vertical solid line in Figure 7) to 0124 UT (the second vertical solid line in Figure 7). Figure 7 (top) is the variation of the $SYM-H$. Figure 7 (bottom) is the integration of the energy
The energy inputs and dissipations vary with time during the major magnetic storm event on 21–22 January 2005. (top) The variation of the SYM-H index. (bottom) The black solid line is the energy input calculated from the energy coupling function from this study, the dotted line is for the $\varepsilon$ function, and the dashed line is the energy dissipation in ring current and auroral ionosphere.

Figure 7. The energy inputs and dissipations vary with time during the major magnetic storm event on 21–22 January 2005. (top) The variation of the SYM-H index. (bottom) The black solid line is the energy input calculated from the energy coupling function from this study, the dotted line is for the $\varepsilon$ function, and the dashed line is the energy dissipation in ring current and auroral ionosphere.

5.5. Energy Dissipation

In general, the energy transferred into the magnetosphere will be dissipated through Joule heating, ring current injection, particle precipitation, and plasmoidejection in the tail. Lu et al. [1998] and Tanskanen et al. [2002] conclude that the ionospheric energy dissipation is dominant. Li et al. [2012] found that the proportion of high-latitude ionospheric dissipation decreases as the storm intensity increases. Furthermore, Kamide and Baumjohann [1993] argue that the energy dissipated in the polar ionosphere is about one third of total energy input into the magnetosphere from the solar wind. The energy dissipation investigation is not the purpose of this paper. We only give a rough estimation of the energy dissipated by Joule heating and precipitation.

Joule heating, calculated from the scalar product of the current and electric field, is a term used to describe the Ohmic production of heat. In the ionosphere, the Joule heating $P_{JH}$ is calculated as $P_{JH} = \int \mathbf{J} \cdot \mathbf{E} \, dS = \int \Sigma \mathbf{E}^2 \, dS$. $\mathbf{J}$ is the electric current, and $\mathbf{E}$ is the electric field imposed on the ionosphere $dS$ in the area element on the spherical ionospheric surface. Here we neglect the velocity of the thermospheric wind because of the difficulty of obtaining global measurements of the neutral winds.

The results indicate that about 23% of the energy input is dissipated via Joule heating. Akasofu [1981] states that the electron precipitation was about half of the ionosphere Joule heating. Østgaard et al. [2002a] conclude that the energies dissipated in the ring current, via Joule heating and by precipitation are 15%, 56%, and 29% of the total energy dissipation, close to the result of Knipp et al. [1998] (17%, 60%, 23%). The ratio of the integrated Joule heating to the integrated precipitation was similar with the result of Lu et al. [1998]. Ahn et al. [1983] find that the average ratio is 4, as they underestimate the precipitation by a factor of about 3 [Østgaard et al., 2002b] and used a constant value of Joule heating. Richmond et al. [1990] found the ratio to be about 3, but they may underestimate the precipitation by about 30% [Østgaard et al., 2002b]. Based on these previous studies of the energy budget in the magnetosphere, we estimate the energy dissipation via auroral particle precipitation to be 12% of the energy input. Therefore, the energy dissipation input from both energy coupling functions and the energy dissipation. The dashed line in Figure 7 (bottom) denotes the energy dissipation via ring current and high-latitude ionosphere using the empirical equations from Akasofu [1981] and Østgaard et al. [2002a, 2002b]. The dotted line is derived from the $\varepsilon$ function. The solid line is the integrated energy input from our energy coupling function. During the main phase, the integrated energy input derived from the $\varepsilon$ function was constant until 0140 UT due to northward IMF $B_Z$, while the energy dissipation increased gradually. The slope of energy input derived from the $\varepsilon$ function was nearly zero, indicating that the net energy input was nearly zero for the northward IMF. However, the energy dissipation via the ring current and ionosphere increase gradually and become larger than the energy input from the $\varepsilon$ function. Du et al. [2008] explain this excess dissipation as due to solar wind energy stored in the magnetosphere in the form of magnetic energy and then converted to the kinetic energy of particles in the ring current during the main storm phase. They called this period the energy releasing phase. However, the epsilon parameter was designed to estimate the energy input into the inner magnetosphere, so it cannot be used to estimate the energy input into all the magnetosphere regions. It underestimates the energy input, especially when the IMF is northward. The energy input calculated from our energy coupling function gives a significant energy input into the magnetosphere even during pure northward IMF conditions which can supply the energy dissipated via the ring current. The energy dissipation via the ring current increases along with the increase in the energy input.
in the polar ionosphere is estimated to be about 35% of the energy input, consistent with Kamide and Baumjohann [1993].

The average input efficiency \( \left( \frac{\text{Energy Input}}{\text{Solar Wind Kinetic Energy}} \times 100\% \right) \) is also estimated to be about 13%, which is close to the results of Li et al. [2012], 11.8% during the main phase of intense storms and 6.2% during the entire storm. Here we use the maximum cross section to calculate the solar wind kinetic energy. Considering that the energy input in this study is not only dissipated in the inner magnetosphere but also dissipated in tail, the input efficiency may be larger than in previous studies which used the \( \epsilon \) parameter to estimate the energy input into the inner magnetosphere.

This study is mainly focused on the energy input calculation from the MHD simulation and on the parameterizations. All above results are based on the simulations with a uniform Pedersen conductance in the ionosphere. The conductance model of the simulation model could be improved with nonuniform empirical conductance models.

6. Summary

We investigate the energy input from the solar wind into the magnetosphere as a function of interplanetary and solar wind conditions based on the PPMLR-MHD simulations. We calculate the energy input for 37 simulation cases with the IMF clock angle changing from 0° to 360°, with other solar wind parameters set to typical values \( (V = 400 \text{ km/s}, n = 5 \text{ cm}^{-3}, B_z = -5 \text{ nT}) \), to fit the expression of \( G(\theta) \) in the dimensional analysis result by Vasyliunas et al. [1982]. A total of 240 simulation cases with different combinations of the solar wind conditions are performed to nonlinearly fit the formula of the total energy input, given as

\[
E_{in} = 3.78 \times 10^7 n^{0.24} V^{-1.47} B_T^{0.86} \left[ \sin^2 \left( \frac{\theta}{2} \right) + 0.25 \right] \text{[W]} \tag{14}
\]

where \( n \) is the number density in \( \text{cm}^{-3} \), \( V \) is the solar wind velocity in \( \text{km/s} \), and \( B_T \) is the transverse IMF in \( \text{nT} \). The energy input \( E_{in} \) includes two parts: one part is a sine power function controlled by the IMF clock angle and the other is a constant independent of the IMF clock angle. The first part is the electromagnetic energy input involving the dayside magnetic reconnection and the second part is the energy input caused by other processes, such as high-latitude magnetic reconnection. The energy input represented by the second part includes both the mechanical energy input and the electromagnetic energy input. When the IMF is purely southward, the energy input is mainly from electromagnetic energy coupling; when the IMF is northward, the mechanical energy input will be dominant. In addition, the power law exponents of the solar wind velocity and \( \sin(\theta/2) \) are larger than the other parameters, implying that the energy input is more sensitive to the solar wind velocity and the IMF clock angle than other parameters.

To validate our energy coupling function inferred from MHD simulations and dimensional analysis, we studied the correlations between the coupling functions including the \( \epsilon \) function, \( E_{in} \) with seven magnetospheric activity indices and the hemispheric auroral power. The results indicate that the energy coupling function found in this study performs better than the \( \epsilon \) function. The effect of the grid resolution and the error of the streamline method for determining the magnetopause location are also tested. The energy dissipation in the ionosphere and the energy input efficiency are estimated to be about 35% of the energy input and about 13% of the solar wind kinetic energy, which are consistent with previous results. Nevertheless, the widely used \( \epsilon \) is designed to estimate the energy input into the inner magnetosphere, and it is thus inappropriate to use it to estimate the energy input into all the magnetospheric regions. It underestimates the energy input, especially during northward IMF conditions. The energy dissipation and the effect of the conductance will be investigated in detail in the future.

References


