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# Spatial Distribution and Anisotropy of Energetic Particles Accelerated by Shock Waves: Focused Transport Model *

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*The focused transport equation (FTE) contains the necessary physics of shock acceleration but avoids the limitation of small pitch-angle anisotropy inherent in the cosmic ray transport equation. We present a focused transport model based on FTE to investigate the spatial distribution and anisotropy of energetic particles accelerated by shock waves from pickup ions. It is found that in the upstream of the shock, the accelerated particles are highly anisotropic, but in the downstream it is approximately isotropic. An intensity spike is formed across the shock. These simulation results are qualitatively consistent with the observations of the termination shock particles by two Voyager spacecraft in the vicinity of the termination shock.*

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The Voyager 1 & 2 spacecraft crossed the termination shock (TS) in 2004 and 2007 respectively, which was a great milestone for mankind on journey to interstellar space. In the vicinity of TS, Voyager's observations of energetic particles with energy below ∼20 MeV,[1] which are usually named termination shock particles (TSPs), show a number of interesting and unexpected features. First, in the upstream of the TS, the ions pitch-angle anisotropies are very large, but in the downstream in the heliosheath, it converges to a steady-state isotropic distribution.[2,3] Second, there appears to be a large highly anisotropic intensity spike.[3] Third, the energetic ion spectrum tends to have multiple power-laws[4] contrasted with the expected single power law that is often observed associated with the energetic particles in the heliosphere. Some of these features of TSPs still await satisfactory explanations.

The most commonly accepted theory for particle acceleration at shock waves is diffusive shock acceleration (DSA). The basic assumption of the DSA theory is that the distribution function of particles is isotropic to first order so that the diffusion approximation could be satisfied.[5,6] Under this limitation, conventionally, the acceleration process is governed by the cosmic ray transport equation (also called the Parker equation)[7] that is the solution of the pitch-angle-averaged distribution function, which equivalently requires that particle velocities measured in the local plasma frame are large enough compared to the plasma flow speed in the shock frame. That is to say, the standard DSA theory only investigates the dynamics of already efficient high-energy particles near the shock. It is commonly believed that the termination shock is dominated by pickup ions (PUIs) and PUIs are the source particles that can be accelerated by TS to be responsible for the formations of TSPs and anomalous cosmic rays (ACRs)[8] that are another type of energetic particles with higher energy. Pickup ions originate from the ionization of the penetrated interstellar neutral atoms and distribute everywhere from the region near the sun to the termination shock. The newborn pickup ions have energy of about 1 keV in the plasma frame. For such low-energy particles, obviously, the DSA theory is not adequate to describe their acceleration process. As a matter of fact, for example, ion intensities peaked sharply at the TS and subsequently dropped during the Voyager crossing, i.e. the intensity spike[2] contradicts with the standard DSA theory where phase space density should conserve across the shock.

In recent years, there is a trend to look at the shock acceleration through the focus transport equation (FTE).[9−11] FTE is the full transport equation that describes the evolution of the gyrotropic distribution function $f(x,\ \boldsymbol{v},\ \mu)$ of energetic particles as space position $x$, particle speed $v$, and the pitch angle cosine $\mu = \cos\theta$.[12−15] It treats the interaction of an energetic particle with the magnetic irregularities as scattering and adiabatic focusing in the pitch angle. FTE contains the principal physical mechanisms accelerating particles including first-order Fermi acceleration, drift acceleration, as well as second-order Fermi acceleration in the turbulent downstream.[91,12] The equation is capable of solving the evolution of the particle in $\mu$ space with no restriction, allowing the particle distribution function to be highly anisotropic so that the isotropy limitation of the DSA is naturally broken. Therefore, it is potentially efficient to use the FTE to describe the acceleration process of low-energy particles which probably produce anisotropic distribution. We have recently developed a one-dimensional focused transport model based on the stochastic differential approach. The energy spectra of energetic particles accelerated by shock waves have been analyzed in our recent paper based on this model.[15] It is proved that the focused transport theory is essentially an extension of the standard DSA theory and the fo-

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cused transport model is a powerful tool applicable to modeling the acceleration of low-energy particles at shocks with arbitrary obliquity.[9,15] In this Letter, we present its application to the pickup ions acceleration at termination shock, focused on the spatial distribution and the anisotropy of energetic particles which have drawn relatively little attention to date in the shock acceleration research community. In what follows, we first briefly introduce the one-dimensional focused transport model and then show the simulation results.

For simplicity, we treat the termination shock as a one-dimensional planar shock with the assumption that the distribution function only depends on the spatial coordinate along the shock normal direction. With a single spatial dimension, FTE runs as follows:

$$\frac{\partial j}{\partial t} = \Big[\frac{\partial^2}{\partial \mu^2}D_{\mu\mu} - \frac{\partial}{\partial x}(u + v\mu\cos\psi) - \frac{\partial}{\partial p}\Big(\frac{dp}{dt}\Big) - \frac{\partial}{\partial \mu}\Big(\frac{\partial D_{\mu\mu}}{\partial \mu} + \frac{d\mu}{dt}\Big)\Big]j + Q(x,p,\mu), \tag{1}$$

$$\frac{dp}{dt} = -p\frac{du}{dx}\Big(\frac{1-\mu^2}{2}\sin^2\psi + \mu^2\cos^2\psi + \frac{\mu u}{v}\cos\psi\Big), \tag{2}$$

$$\frac{d\mu}{dt} = \frac{1-\mu^2}{2}\frac{du}{dx}\Big(\frac{v}{u}\sin^2\psi\cos\psi + \mu(1-3\cos^2\psi) - 2\frac{u}{v}\cos\psi\Big), \tag{3}$$

where $j$ is the particle energy differential flux related to the gyrotropic distribution function by $j = 4\pi p^2 f$. Here the variables of the momentum $p$ and the pitch angle cosine $\mu$ are defined in the local solar wind plasma frame while the spatial coordinates $x$ are defined in a fixed frame, i.e. the equation is built in the mixed frame. $\Psi$ is the angle between the magnetic field and the shock normal. $Q(x,p,\mu)$ is the source term of particle injection. The newborn pickup ions in the vicinity of termination shock tend to form an isotropic shell distribution in the ambient solar wind frame due to the scattering by the ambient Alfvénic waves. Accordingly, isotropic pickup protons with mono-energy (in approximation) are continuously injected at the termination shock for acceleration, i.e. $Q(x,p,\mu) = Q_0\delta(p-p_0)\delta(x)$. $Q$ is the injection rate. In this study, the acceleration process of such particles by the termination shock is simulated. Note that here the perpendicular diffusion is neglected since it is much smaller relative to the parallel diffusion that arises from pitch angle scattering in the environment of termination shock vicinity.[16] We define the $X$ axis, which denotes the unique spatial coordinates, along the shock normal from downstream to upstream, and the coplanar plane of the plasma flow and magnetic fields as the $x$–$y$ plane. The plasma flow is normally incident ($U = ue_x$). We model the compressional flow in the thin ramp transition simply by using a linear profile. The introduced transport equation is a three-dimensional parabolic differential equation. In our model, it is solved through a stochastic differential approach based on the mathematical equivalence between the Fokker–Planck diffusion problem and the system of stochastic differential equations (SDEs).[13,15,17,18] A Monte-Carlo simulation based on the stochastic approach is conducted with millions of pseudo particles injected. Finally, the numerical solution is normalized with $Q_0$ equal to 1. For the details of this method, please refer to Ref. [15].

In accordance with the Parker model, the shock obliquity $\theta_{Bn}$, i.e. the angle between the shock normal and the upstream magnetic field direction, should be 89.4°. In actual Voyager 1 observations, the upstream magnetic field near the termination shock varies with time in a broad range. The average magnetic field direction is nearly perpendicular to the radial direction, as predicted by the Parker model. However, in a fraction of time, the angle between the magnetic field and the radial direction is less than 60° and is sometimes even zero. Considering the termination shock configuration change, parallel, oblique, and quasi-perpendicular shocks may contribute to the pickup ions acceleration. In this study, pickup ion accelerations at parallel shock, oblique shock and quasi-perpendicular shock are simulated by taking a fixed $\theta_{Bn}$ equal to 0°, 45°, 80°, respectively, (named cases 1, 2 and 3). The shock strength (compression ratio) is given as $s = 3.7$. For parallel shock and oblique shock, the mono-energetic injection speed of pickup ions is set as 400 km/s, which is equal to the upstream solar wind speed. It corresponds to the injection of newborn pickup ions. For a highly oblique shock, once transmitted upstream to downstream, the particles cannot easily return to the shock since the magnetic field is nearly perpendicular to the shock normal. Thus, with no multiple shock-crossings the newborn pickup ions cannot be efficiently accelerated by highly oblique quasi-perpendicular shock, which has been verified by Zuo *et al.*[15] Thus here we consider an injection of pre-accelerated pickup ions with speed $v = 1.5u_1/\cos\theta_{Bn}$ (larger than the de Hoffmann-Teller speed) for the discussed quasi-perpendicular shock acceleration.

Figure 1 shows how the spatial density of the accelerated energetic particles at all energies evolves in the shock vicinity for each simulated case introduced above. In the simulation, the maximum energy of the particles, which is set as one of the calculation boundaries, is given as $T_{\max} = 10$ MeV for cases 1 and 2 where the injection energy is around 0.83 keV, and $T_{\max} = 40$ MeV for case 3 with a higher injection energy. The distribution function value is normalized with the averaged value in the downstream region set as 1. Qualitatively, the three curves appear to have very similar features. In the upstream region (corresponding to negative $X$) the intensity of particles increases exponentially toward the shock. Then there is an abrupt increase in intensity across the shock transition, after which it drops to a plateau form in the downstream region, i.e. it behaves as a typical intensity spike structure. In the downstream region, the intensity is nearly constant except for some fluctuations. The intensity drop is relatively weak for parallel shock acceleration, and the drop height increases with increasing shock obliquity. The standard DSA theory assumes that the direction-averaged particle distribution function is continuous in space across the shock due to the isotropy limitation so that it cannot be used to explain the formation of intensity spike structure observed by Voyagers near the TS. The diffusive theory requires that all the injected particles in the upstream are transmitted to downstream with energy gain on average due to the Fermi acceleration. This is valid for parallel shock acceleration. However, for non-parallel shocks for which the magnetic field magnitude and direction change abruptly within the ramp, a portion of particles could be reflected back to upstream due to the magnetic kinks. The reflections are expected to lead to the discontinuity of the distribution function and produce "reflection peak".[19] According to an analytical solution, Gieseler *et al.*[19] pointed out that, if the obliquity of the shock induces anisotropy, a density jump would be produced by the particle reflections. The reflection efficiency depends on the shock obliquity and particle energy. In other words, the induced density drop is related to the shock obliquity and the injection speed. In terms of the focused transport theory, the focusing of a non-uniform magnetic field is one of the important factors that determine the pitch angle change rate (see Eqs. (2)–(5) in Ref. [15] and the explanations therein). The reflection effect is in fact incorporated in the focused transport equation. Our simulation results show that the intensity spike is a natural result of low-energy particle acceleration by oblique shock (cases 2 and 3). Here there is also a small intensity drop across a parallel shock in Fig. 1(a). It is known that no particle reflection takes place for parallel shock acceleration if the cross-shock potential is not considered. Different from that for the oblique shock, it is due to the fact that the distribution function is measured in the mixed system.[9] Le Roux *et al.* pointed out that the intensity is still continuous across the parallel shock if it is transformed to the shock frame.[9]

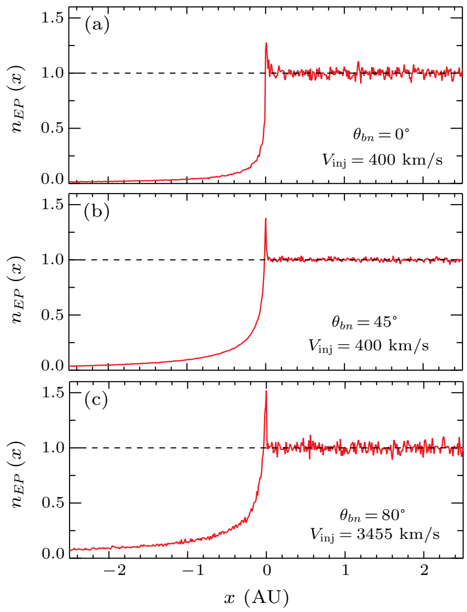


**Fig. 1.** The spatial density distribution of the energetic particles accelerated by different types of shocks from pickup ions: (a) parallel shock with $\theta_{Bn} = 0°$; (b) oblique shock with $\theta_{Bn} = 45°$; (c) quasi-perpendicular shock with $\theta_{Bn} = 80°$.

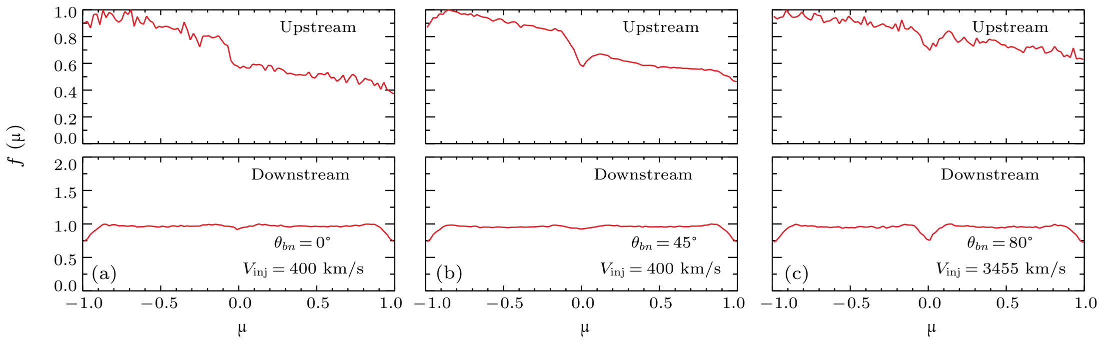


**Fig. 2.** The pitch angle distribution in the upstream and downstream regions of the energetic particles accelerated by different types of shocks from pickup ions: (a) parallel shock with $\theta_{Bn} = 0°$; (b) oblique shock with $\theta_{Bn} = 45°$; (c) quasi-perpendicular shock with $\theta_{Bn} = 80°$.

Figure 2 presents the pitch angle distribution (PAD) of accelerated particles in the upstream and downstream regions for the three cases, which is obtained from the statistical analysis for all the particles at all energies. Here we do not show the distribution function at a certain energy and discuss the energy dependence of the anisotropy because of the poor statistics in the Monte–Carlo simulation based on the stochastic approach. The distribution function is normalized with the maximum of $f(\mu)$ set as 1. For different types of shock acceleration, the PAD features are similar. In the upstream region, the energetic particles are highly anisotropic. It can be seen that the distribution function reaches its maximum at about $\mu = -1$. This means that most particles in the plasma frame move along the field in the upstream direction away from the shock (the magnetic field is assumed to point in the downstream direction). There is an obvious transition of $f(\mu)$ from $\mu < 0$ toward $\mu > 0$. In the downstream region, the particles are nearly isotropic. The distribution function is approximately the same for all pitch angles except some deviations near the position with $\mu = 0$, 1, 1. The simulation results are qualitatively consistent with the Voyager anisotropy observations of TSPs. Before Voyager 1 crossed the termination shock, it observed the tendency toward upstream field-aligned beaming away from the shock, but in the far downstream, the TSPs approached an isotropic status.

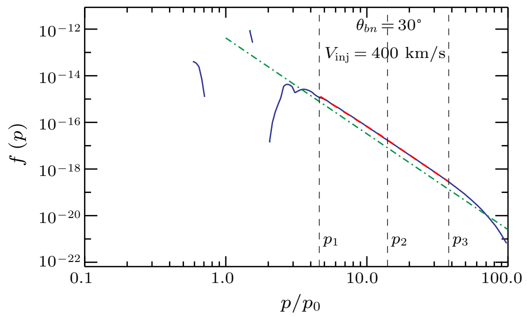


**Fig. 3.** An example of the energy spectrum of the energetic particles in the downstream region accelerated by oblique shocks. Here the shock obliquity is set as $\theta_{Bn} = 30°$.

The observations of energetic ions by the two Voyager spacecraft in the vicinity of the termination shock provide us with essential information about pickup ion acceleration. In the above discussions it can be seen that the focused transport model can reproduce the large-scale observational features of the anisotropies and the spatial distribution of accelerated pickup ions. The energy spectrum observations can also be qualitatively explained with the focused transport model. Cummings *et al.*[4] found that the energetic proton tends to form a spectrum of exceptional four power laws that may be evidence of two-component acceleration: at low energies the energy spectrum is a double power-law spectrum with a break at 0.4 MeV; and a second component appears above 1 MeV, consisting of two power-law spectra with a break at 3.2 MeV. Our previous test simulation study has shown that the accelerated energy spectrum of pickup ions acceleration by oblique shock is a double power-law spectrum, as shown in Fig. 3(a) and Fig. 6(a) in Ref. [15]. Here, for completeness, we present another example of energy spectra for the case with shock angle $\theta_{Bn}$ equal to 30° in Fig. 3. Other parameters in the simulation are set to be the same as those in cases 1–3. The solid line is the simulated spectrum and the dash-dotted line is the theoretical solution of the Parker equation presented here for comparison. It can be clearly seen that at higher energies it is a double power-law spectrum, i.e. a harder power-law spectrum (see the red fitted line between $p_1 = 5p_0$ and $p_2 = 15p_0$) followed by another power-law spectrum with a slope the same as the spectral index given by DSA solution (see the red fitted line between $p_2 = 15p_0$ and $p_3 = 38p_0$). As we know, the termination shock has an obliquity of ∼89.4° in large scale. The magnetic field observations of Voyager 1 near the termination shock indicate that the resulting shock obliquity deviates from the average angle of 89.4° to less than 60° in a significant fraction of time, i.e., this shock is no longer always a quasi-perpendicular shock. During the magnetic field deviation time interval, pickup ions are expected to be accelerated to several or even tens of times the original speed in the solar wind frame since the shock obliquity $\theta_{Bn} < 60°$. Some of the particles are accelerated to high energies to form a double power-law spectrum. The pre-accelerated particles, whose speed is beyond the injection threshold (around the de Hoffmann–Teller speed $V_{\rm dHT} = u_1 \sec\theta_{Bn}$), are able to be further accelerated by the highly oblique termination shock to higher energy. Hence, at the very high energies, the spectra resemble another double power-law spectrum. Eventually, the accelerated energy spectra should include two parts corresponding to two components of acceleration, respectively, with a transition and four power laws generated.

In summary, we have applied a focused transport model to study the pickup ion acceleration at the termination shock. The simulation results show that this model is able to reproduce some major observational features of the low-energy TSPs, such as the upstream anisotropic distribution and the intensity spike near the shock, which cannot be understood from the standard DSA theory. The observed multiple power-law energy spectra can also be qualitatively explained with this test simulation. For our current one-dimensional model, it is assumed that the plasma and magnetic fields in the vicinity of the termination shock are stable, and the termination shock is also modeled as a stable planar shock. It can reveal the physical contents hidden in the observational phenomena. However, to reproduce the Voyager observations, the time-varying of the background environment, the shock fine structures as well as the nonlinear feedback of the accelerated energetic particles on the geometry, etc., need to be considered. This calls for a time-dependent multi-dimensional focused transport model. In addition, less energetic particles of solar origin and pickup ions have a vital role as seeds of energetic particles accelerated in the heliospheric environment. However, the traditional DSA theory is not adequate to describe the acceleration process of low-energy particles. The focused transport model has the potential capability of investigating the injection and acceleration of such particles as an extension of the DSA theory since it is able to naturally remove the isotropy limitation. Our next consideration is devoted to developing focused transport models to study the SEP injection, pickup ions acceleration in the corotating interaction region and other energetic particle phenomena.

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