Basic Space Plasmas Physics
Assignment 3

T. Chen*
J. G. Huang†
W. Y. Li‡ (TA)
Center for Space Science and Applied Research, CAS.

September 30th, 2011

(Solve three out of the four problems! If time permits, you can solve all.)

1 Earth's Magnetic Field

Earth's magnetic field is approximately a magnetic dipole, with the magnetic field S pole near the earth's geographic north pole and the other magnetic field N pole near the earth's geographic south pole. The scalar potential of the earth's dipole is given by

$$U = \frac{\mu_0}{4\pi}\frac{\vec{M}\cdot\vec{r}}{r^3} \tag{1.1}$$

where μ_0 is the permeability of free space, $\vec{M} = -M\vec{e}_z$ is the dipole moment, and r is the distance from the center. Earth's magnetic field is given by

$$\vec{B} = -\nabla U \tag{1.2}$$

Derive earth's magnetic field is given by

$$B_r = -\frac{\mu_0 M}{2\pi}\frac{\sin\lambda}{r^3} \tag{1.3}$$

$$B_\lambda = \frac{\mu_0 M}{4\pi}\frac{\cos\lambda}{r^3} \tag{1.4}$$

$$B_\phi = 0 \tag{1.5}$$

where λ is magnetic latitude.

Hint: Use the result of Assignment 2, Question 1.

2 Earth's Magnetic Field Lines

Show that the equation for the earth's magnetic field line is $r = r_0\cos^2\lambda$ and write down expression for the magnitude of the magnetic field $B(r,\lambda)$. Show the ratio of the magnetic field to that at the equator B_0 is

$$\frac{B}{B_0} = \frac{(1+3\sin^2\lambda)^{1/2}}{\cos^6\lambda} \tag{2.1}$$

*tchen@cssar.ac.cn
†huangjg@cssar.ac.cn
‡wyli@spaceweather.ac.cn

3 Drift in Earth's Magnetic Field

If the magnetic field is characterized by both gradient and curvature terms in $\nabla\vec{B}$ and there are no currents present so that $\nabla \times \vec{B} = 0$, then the total drift velocity is given by

$$\vec{v}_B = (v_\parallel^2 + v_\perp^2/2)[(\vec{B} \times \nabla B)/(\omega_g B^2)] \tag{3.1}$$

where $\omega_g = \dfrac{qB}{m}$ is the gyrofrequency or cyclotron frequency.

Show the total drift velocity of a particle at the dipole's equator is given by

$$\vec{v}_B = -\frac{3mv^2}{2qBr}\vec{e}_\phi \tag{3.2}$$

where $\vec{e}_\phi$ is the local orthogonal unit vector in the direction of increasing ϕ.

4 Particle Trapped in Earth's Magnetic Field

Earth's magnetic field can trap charged particles such as electrons and protons as they are forced to execute a spiraling motion back and forth along the field lines (Figure 1). At the equator,

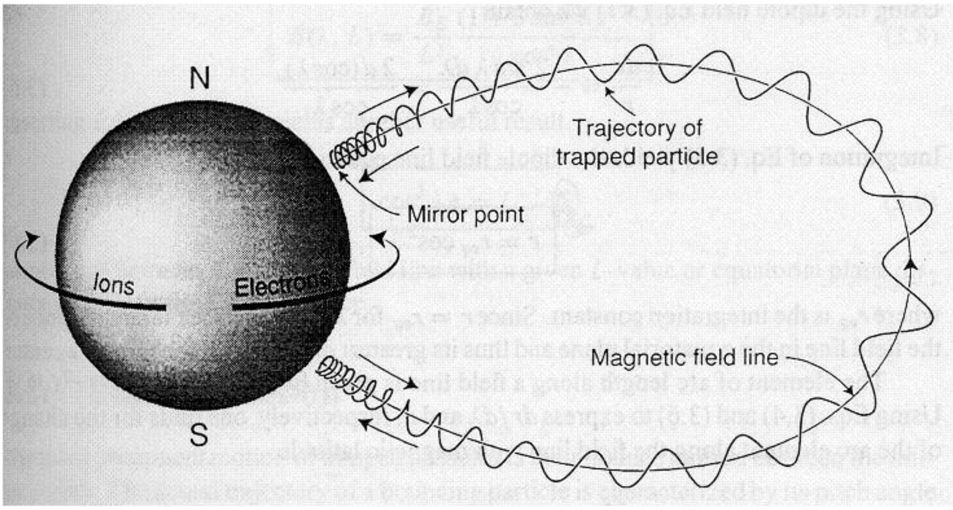

Figure 1: Mirror motion of charged particle in Earth's magnetic field

there is loss cone for the charged particles, and particles within the loss cone will hits the Earth's surface before mirroring back. Try to show that the boundary of the loss cone is at

$$\sin^2\alpha_0 = \frac{1}{(4L^6 - 3L^5)^{1/2}} \tag{4.1}$$

and make a schematic to illustrate the profile of α_0 versus L. Here, α_0 is the pitch angle of particles at the equator and L is the L-value of Earth's magnetic field.