Basic Space Plasmas Physics
Assignment 6

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(Solve two out of the three problems! If time permits, you can solve all.)

1 Oblique Alfven Wave

The ideal MHD equation for magnetized plasma is given by

$$\rho_0 \frac{\partial V_1}{\partial t} = \frac{1}{\mu_0}(\nabla \times \vec{B}_1) \times \vec{B}_0 \tag{1.1}$$

$$\frac{\partial \vec{B}_1}{\partial t} = \nabla \times (\vec{V}_1 \times \vec{B}_0) \tag{1.2}$$

where ρ_0 is the mass density, $\vec{V}_1$ and $\vec{B}_1$ are the small perturbation of the bulk velocity and the magnetic field, respectively, μ_0 is the permeability in vacuum.

The incompressible plasma is initially in stationary equilibrium state with uniform density ρ_0 and magnetic field $\vec{B}_0$. Show the dispersion relation of the oblique Alfven wave as

$$\omega^2 = \frac{(\vec{k} \cdot \vec{B}_0)^2}{\mu_0 \rho_0} \tag{1.3}$$

where ω is the wave frequency and $\vec{k}$ is the wave vector.

2 Upper Hybrid Electrostatic Wave

Consider the plasma is in high-frequency motion, only electron moves through a isothermal process $T_e = constant$, and magnetic field $B_0\vec{e}_z$ does not change (non-magnetized plasma). A plane wave where all perturbed physical quantities vary as $\exp(\mathrm{i}\vec{k}\cdot\vec{r} - \mathrm{i}\omega t)$ is adopted to study the wave. Wave vector and disturbed electric field are all in x direction. The characteristic thermal speed of the electron is given by $u_{te} = (T_e/m_e)^{1/2}$. Linearizing single fluid MHD equation and Maxwell equation about a stationary equilibrium state with electric field $\vec{E}_0 = 0$ and $\vec{u}_{0e} = 0$, we have

$$\begin{cases} -\mathrm{i}\omega n_1 + \mathrm{i}k n_0 u_{1x} = 0 \\ -\mathrm{i}\omega m_e u_{1x} = -eE_{1x} - eu_{1y}B_0 - \mathrm{i}kT_e n_1/n_0 \\ -\mathrm{i}\omega m_e u_{1y} = eu_{1x}B_0 \\ \mathrm{i}kE_{1x} = -e\dfrac{n_1}{\varepsilon_0} \end{cases} \tag{2.1}$$

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Table 1: Plasma Parameters

	Magnetosphere	Magnetosheath
n_p (m^{-3})	2×10^7	8×10^7
n_e (m^{-3})	2×10^7	8×10^7
$\vec{U}$ (km/s)	(0, 0, 0)	(150, 0, 0)
$\vec{B}$ (nT)	(0, 0, 30)	(25, 0, 0)

Show the dispersion relation of upper hybrid electrostatic wave.

3 Kelvin-Helmholtz Instability

Use the data shown in the Table 1 to predict whether the Earth's magnetopause is stable or unstable to the growth of Kelvin-Helmholtz waves. Estimate the maximal growth rate and the corresponding wave phase velocity.